

MA 113 CALCULUS I, FALL 2016
WRITTEN ASSIGNMENT #3
Due Friday, September 30, 2016, at the beginning of lecture

Instructions: The purpose of this assignment is to develop your ability to formulate and communicate mathematical arguments. Your complete assignment should have your name and section number on each page, be stapled, and be neat and legible. *Unreadable work will receive no credit.*

You should provide well-written, complete answers to each of the questions. We will look for correct mathematical arguments, complete explanations, and correct use of English. Your solution should be formulated in complete sentences. As appropriate, you may want to include diagrams or equations written out on a separate line. You may read your textbook to find examples of how we communicate mathematics.

Students are encouraged to use word-processing software to produce high quality solutions. However, you may find that it is simpler to add graphs and equations using pen or pencil.

Consider the graph of the parabola given by the function $f(x) = x^2$ and let (a, b) be any point in the plane. The problem is to decide how many lines, if any, can be drawn from the point (a, b) so that the lines are tangent to the graph of the parabola.

1. (2 points) Draw the graph of $f(x)$. Take some sample points P and try to draw a line from P that is tangent to the parabola at some point. Make a conjecture that describes when this can be done and when it cannot be done. Then refine your conjecture to guess how many tangent lines to the parabola can be drawn from a given point P .
2. (2 points) Let (r, r^2) be an arbitrary point on the parabola $f(x) = x^2$. Show that the equation of the tangent line to the graph of $f(x)$ at (r, r^2) is given by $y = 2rx - r^2$.
3. (1 points) Let $P = (a, b)$ be an arbitrary point in the plane. Show that a line can be drawn through P that is tangent to the parabola precisely when one can find a real number r such that (a, b) lies on the line $y = 2rx - r^2$.
4. (1 points) Show that you must solve the quadratic equation $r^2 - 2ar + b = 0$.
5. (2 points) Use the quadratic formula to solve for r to show that such a line exists precisely when $b \leq a^2$.
6. (2 points) Put everything together to state precisely when there are no tangent lines, exactly one tangent line, or exactly two tangent lines that can be drawn from P to the parabola.