MA 113 Calculus I, Fall 2016
Written Assignment #4
Due Friday, October 7, 2016, at the beginning of lecture

Instructions: The purpose of this assignment is to develop your ability to formulate and communicate mathematical arguments. Your complete assignment should have your name and section number on each page, be stapled, and be neat and legible. *Unreadable work will receive no credit.*

You should provide well-written, complete answers to each of the questions. We will look for correct mathematical arguments, complete explanations, and correct use of English. Your solution should be formulated in complete sentences. As appropriate, you may want to include diagrams or equations written out on a separate line. You may read your textbook to find examples of how we communicate mathematics.

Students are encouraged to use word-processing software to produce high quality solutions. However, you may find that it is simpler to add graphs and equations using pen or pencil.

An important application of calculus is the approximation of square, cube, and other roots of numbers. For example, $\sqrt{2}$ is the only positive solution of $x^2 - 2 = 0$. $\sqrt[7]{5}$ is a solution of $x^7 - 5 = 0$. In this assignment, we will use *Newton's Method* to approximate roots; to learn more about Newton's method, read section 4.8 in your textbook.

1. (2 points) Consider $f(x) = x^2 - 2$. Suppose you make an initial guess that the solution to $f(x) = 0$ is $x = 7$. This is obviously a ridiculous guess, since $7^2 - 2 = 47$, but it's okay because everyone has a bad day now and then. To move from your guess to something closer to the correct solution, set $x_1 = 7$ and successively find the following values:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}, \qquad x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}, \qquad x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

   Write down the *fractions* for x_2, x_3, and x_4 as your answer to this problem — *do not write down decimals here.*

2. (2 points) Continue using the recursion $x_n = x_{n-1} - \dfrac{f(x_{n-1})}{f'(x_{n-1})}$ for $n = 5$ and $n = 6$.

   Write down the *fractions* for x_5 and x_6. Type your value of x_6 into WolframAlpha (at www.wolframalpha.com) and compare it to the value of "sqrt(2)" given by WolframAlpha. How many digits do these two numbers agree to?

3. (2 points) Set $g(x) = x^7 - 5$. Using the initial guess of $x_1 = 4$ and the recursion

$$x_n = x_{n-1} - \frac{g(x_{n-1})}{g'(x_{n-1})}, \tag{1}$$

   iterate the recursion until x_n agrees with $\sqrt[7]{5}$ to two decimal points, as given by entering "seventh root of 5" into WolframAlpha. *Show your work.* What is the value of n that you need to get two digits of accuracy?

4. (4 points) Approximate $\sqrt[3]{6}$ by identifying a polynomial $h(x)$ such that $h(\sqrt[3]{6}) = 0$ and using the same recursion as above with the initial value $x_1 = 5$. Stop when your approximation matches four decimal points when compared to the value given by entering "cube root of 6" into WolframAlpha. *Show your work.* What value of n did you need in order to reach this level of accuracy?