

MA 113 CALCULUS I, FALL 2017
WRITTEN ASSIGNMENT #4
Due Friday, October 6, 2017, at beginning of lecture

Instructions: The purpose of this assignment is to develop your ability to formulate and communicate mathematical arguments. Your complete assignment should have your name and section number on each page, be stapled, and be neat and legible. *Unreadable work will receive no credit.*

You should provide well-written, complete answers to each of the questions. We will look for correct mathematical arguments, complete explanations, and correct use of English. Your solution should be formulated in complete sentences. As appropriate, you may want to include diagrams or equations written out on a separate line. You may read your textbook to find examples of how we communicate mathematics.

Students are encouraged to use word-processing software to produce high quality solutions. However, you may find that it is simpler to add graphs and equations using pen or pencil.

1. (2 points) Let $f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$. Explain why

$$a_0 = f(0), \quad a_1 = f'(0), \quad 2 \cdot a_2 = f''(0), \quad 3 \cdot 2 \cdot a_3 = f^{(3)}(0), \quad 4 \cdot 3 \cdot 2 \cdot a_4 = f^{(4)}(0).$$

2. (2 points) For a general polynomial function $f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n = \sum_{k=0}^n a_kx^k$, how are the coefficients a_k related to the higher derivatives of f ? Justify your answer.
3. (3 points) Explain why a non-zero polynomial function $f(x)$ *cannot* satisfy the equation

$$f''(x) = -f(x).$$

Why does this show that neither $\sin x$ nor $\cos x$ is a polynomial?