

MA 113 CALCULUS I, FALL 2017
WRITTEN ASSIGNMENT #6
Due Friday, November 3, 2017, at beginning of lecture

Instructions: The purpose of this assignment is to develop your ability to formulate and communicate mathematical arguments. Your complete assignment should have your name and section number on each page, be stapled, and be neat and legible. *Unreadable work will receive no credit.*

You should provide well-written, complete answers to each of the questions. We will look for correct mathematical arguments, complete explanations, and correct use of English. Your solution should be formulated in complete sentences. As appropriate, you may want to include diagrams or equations written out on a separate line. You may read your textbook to find examples of how we communicate mathematics.

Students are encouraged to use word-processing software to produce high quality solutions. However, you may find that it is simpler to add graphs and equations using pen or pencil.

1. (2 points) Find $\lim_{x \rightarrow 0} \frac{e^x - 1 - x - \frac{x^2}{2} - \frac{x^3}{6}}{x^4}$.
2. (2 points) Recall that $n!$ is defined by $n! = n(n-1)(n-2) \cdots 3 \cdot 2 \cdot 1$. Thus $1! = 1$, $2! = 2$, $3! = 6$, $4! = 24$ and $5! = 120$. By convention, we set $0! = 1$. Use the calculations in (1) to help you determine the following limit.

$$\lim_{x \rightarrow 0} \frac{e^x - 1 - \frac{x}{1!} - \frac{x^2}{2!} - \frac{x^3}{3!} - \frac{x^4}{4!} - \cdots - \frac{x^{n-1}}{(n-1)!}}{x^n}.$$

[Note that the case $n = 4$ is the problem in (1). Compute this limit for $n = 1$, $n = 2$, $n = 3$, $n = 4$, and look for a pattern. Check your prediction by computing the limit for the case $n = 5$.]

3. (3 points) Try to use L'Hospital's rule to evaluate $\lim_{x \rightarrow \infty} \frac{x}{\sqrt{x^2 + 1}}$. Why does this method fail for this problem? Find a different method to evaluate this limit.