Course outline, MA 113, Spring 2014

Part A, Functions and limits

§1.1–1.2 Functions, domain and ranges, A1.1-1.2-Review (9 problems)

- Functions, domain and range
- Domain and range of rational and algebraic functions
- Linear and quadratic functions
- Common models, distance traveled, geometric problems
- Exam questions:
  - Find domain and range of a given function
  - Find a formula given a description in words
  - Find a linear function given two points or a point and a slope

§1.3,1.5 Functions and inverse functions, A1.3and1.5-Functions (12 problems)

- Definition of composite function
- Definition of inverse functions
- Relation between graph of a function and its inverse
- Finding a formula for an inverse function
- Relation between domain and range for a function and its inverse
- Exam questions:
  - Find inverse and composite fucntions
  - Find domain and range of inverse functions
  - Graph an inverse function

§1.4,1.5 Trigonometric functions, A1.4-1.5-Trig (11 problems)

- Radian measure
- Definitions of sin and cos by unit circle
- Definitions of tan, sec, csc, and cot
- Pythagorean identities, addition formulae for sine and cosine, double angle formulae

- Definitions of inverse trig functions
- Use of trigonometric functions to describe the relations between sides and angles in a right triangle
- Exam questions:
  * Using identities and the unit circle to find values of trig functions
  * Finding sides and angles in right triangles
  * Evaluating expressions such as $\cos(\arcsin(x))$
  * Using $\cos(x)$ and $\sin(x)$ to describe points on a circle

§1.6 Exponential and logarithm functions, A1.6-Exp-Log (12 problems)

- Definition of the logarithm with base e as the inverse of e^x
- Properties of logarithms
- Write b^x in terms of $e^(x \ln(b))$
- Solving equations involving exponential functions
- We do not cover hyperbolic functions in this course
- Exam questions:
  * Using properties of logarithms to evaluate expressions involving logarithms and exponential functions
  * Solving equations involving the logarithm and exponential function
  * Finding A and k so that $f(t) = Ae^{kt}$ has specified properties.

§2.1–2.2 Tangent and velocity problem, A2.1-2.2-Limits (15 problems)

- Slopes of secant lines, Average velocity
- Using numerical estimates to guess slope of tangent line and instantaneous velocity.
- Relation between one-sided and two sided limits, limits from a graph
- Exam questions:
  * Compute average rate of change, compute average velocity

    * Write instantaneous velocity and slope of a tangent line as a limit.
    * Finding limits from graphs
    * Using one-sided limits to determine if a (two-sided) limit exists
    * Guessing limits from numerical values or table of a function

§2.3 Limit laws/computing limits, A2.3-Computing-Limits (9 problems)

- Limit rules for sums, products, quotients, and powers.
- Exam questions:
    * Evaluating limits using these laws
    * Providing step-by-step justifications for evaluating limits and recognizing when a rule does not apply

§2.4 Limits and Continuity, A2.4-Continuity (15 problems)

- Definition of continuous functions
- Continuity of composite functions
- Evaluating limits of continuous functions by substitution.
- Exam questions:
    * Finding the set where a function is continuous
    * Using continuity to evaluate limits and recognizing when this does not apply
    * Choosing parameters in piecewise functions to guarantee continuity

§2.5 Evaluating limits algebraically, A2.5-Evaluating limits

- Indeterminate forms
- Simplify and use limit laws
- Computing slope of tangent line
- Distinguishing between limits which do not exist and limits where the rule for the limit of a quotient does not apply.
- Exam questions:

    - Evaluating limits where algebraic simplification is needed prior to applying limit laws
    - Evaluating limits that arise in computing instantaneous velocities and slopes of tangent lines

§2.6 Trigonometric Limits, A2.6-Trig-limits

- Squeeze theorem
- Limits of $\sin(x)/x$, $(1-\cos(x))/x^2$
- Algebraic manipulations and trig limits.
- Note that §2.7 on limits at infinity will be covered later.
- Exam questions:
    - State squeeze theorem. Provide a complete statement including hypotheses and conclusion
    - Use squeeze theorem to evaluate limits.
    - Recognize when the squeeze theorem does not apply.

Part B, The derivative

§3.1 The derivative, B3.1-derivative (12 problems)

- Definition of the derivative
- Equation of tangent line
- Derivative of linear functions
- Approximating derivatives
- Exam questions:
  - Computing derivative from definition
  - Finding a tangent line
  - Finding or estimating derivative from a graph

§3.2 The derivative as a function, B3.2-Deriv-Function (15 problems)

- Power rule
- Derivative of e^x and characterization of e by its derivative.
- Leibniz notation
- Continuity of differentiable functions
- Exam questions:
  - Computing derivatives
  - Finding tangent lines
  - Proving a differentiable function is continuous
  - Differentiability of piecewise functions

§3.3 Product and quotient rules, B3.3-Product (11 problems)

- Proof of product rule
- Use of product rule and quotient rules
- Exam questions:
  - Compute derivatives with the product rule
  - State product rule

§3.4 Rates of change 3.4, B3.4-Rates (13 problems)

- Interpretation of derivative of as a marginal rate
- Speed, velocity and acceleration
- Motion under the influence of gravity
- Exam questions:
  - Interpretation of derivative as a marginal rate
  - Describing motion

§3.5 Higher derivatives, B3.5-Higher-Deriv (8 problems)

- Definition of higher derivatives
- Velocity, acceleration
- Derivatives of e^x
- Motion of an object with constant acceleration
- Exam questions:
  - nth order derivatives of e^x, x^r, and xe^x
  - Motion of an object falling under gravity

§3.6 Derivatives of trig functions, B3.6-Trig-Deriv (12 problems)

- Proof using basic trig limits and addition formula
- Use of quotient rule to find derivatives of tan cot sec csc
- Exam questions:
  - nth order derivatives of sin and cos
  - Finding the derivative of tan using quotient rule

§3.7 Chain Rule, B3.7-Chain

- Statement and use of chain rule
- Using chain rule to differentiate a reciprocal
- Understand that the generalized power rule, the exponential rule and the shifting and scaling rules are special cases of the chain rule.
- Exam problems:
  - Statement of chain rule

    - Computing derivatives with chain rule

§3.8–3.9 Derivatives of inverse functions, Derivatives of Exponential and Logarithms, B3.8-3.9-Deriv-Inverse

- Derivative of inverse functions
- Derivative of ln(x)
- Derivative of arcsin(x), arctan(x), and arcsec(x)
- Students should not know the derivative of b^x. They should know to rewrite $b^x = e^{x \ln(b)}$.
- Omit material on hyperbolic functions
- Exam questions:
    - Using formula for derivative of an inverse function
    - Using derivatives as above

§3.10 Implicit differentiation, B3.10-Implicit (11 problems)

- Computing first derivatives by implicit differentiation, omit finding the second derivative by implicit differentiation
- Revisit derivatives of inverse functions
- Exam questions:
    - Find tangent line(s) for a curve defined implicitly
    - Use implicit differentiation to find the derivative of an inverse function

§3.11 Related rates, B3.11-Rel-Rates (14 problems)

- Word problems
- Review formulae for areas and volumes as needed. Students should know the formula for area of plane figures such as triangles, rectangles and circles. Formulae related to three dimensional objects will be provided if needed on exams.
- Relation between rates of changes of sides and angles in right triangle
- Exam questions:

* Problems involving rates of changes for sides of a triangle.
* Rates of changes for quantities related by volume and area formulae

Part C, 10 lectures + 2 Review

§4.1 Linear approximation, C4.1-Lin-Approx

- Definition of the linearization of a function
- Computing approximations of functions
- Definition of absolute error and relative error
- Estimation in applied problems
- Exam problems
  - Find a linear approximation and compute errors
  - Estimation in a word problem, e.g. volume of paint on a sphere

§4.2 Extreme values, C4.2-Extrema (12 problems)

- Existence of extreme values on a closed interval
- Failure to attain extreme values on an open interval
- Fermat's theorem
- Critical points and finding extreme values on a closed interval
- Exam problems:
  - Finding extrema
  - Finding critical points
  - Statement of theorem on existence of extrema on closed, bounded intervals
  - Examples to illustrate the failure of the theorem on a closed interval

§4.3 The mean value theorem and monotonicity, C4.3-MVT

- Statement of mean value theorem
- Estimating functions
- First derivative test for monotonicity
- Exam problems:
  - Finding intervals of concavity

    * Finding distance traveled
    * Statement of mean value theorem
    * Use of mean value theorem for estimating functions

§4.4 Shape of a graph, C4.4-Monot-Conc 11 problems

- First derivative test for local extremes
- Concavity
- Second derivative test for concavity
- Second derivative test for local extrema
- Exam problems:
    * Find intervals of concavity and monotonicity
    * Classify extrema
    * Sketch a graph
    * Match graphs to information about derivatives

§2.7 Limits at infinity, C2.7-Limits-Inf (10 problems)

- Definition
- Limits of rational functions at infinity
- Exam problems:
    * Compute limits at infinity

§4.5 L'Hôpital, C4.5-LHopital

- Limits of e^x/x^n
- Limit of $\ln(1+x)/x$
- Limit of $(1+x)^{1/x}$
- Exam problems
    * Compute limits
    * Recognize when L'Hopital does not apply

§4.7 Optimization, C4.7-Optimize (15 problems)

- Formulating a precise optimization problem

- Criteria for finding extreme values
- Justifying existence of extrema on an open interval using first derivative (if the first derivative changes sign exactly once, then we have an extrema where the first derivative changes sign).
- Exam problems
  * Optimization problems. Be able to explain why you have an extrema

§4.8 Newton's method, C4.8-Newton (13 problems)

- Carry out method numerically and graphically
- Deriving the iteration formula
- Applications of finding roots: critical points or values of an inverse function
- Exam problems:
  * Graphical description of Newton's method
  * Find roots
  * Use Newton's method to solve interesting equations. For example to find a critical point or a value of f^{-1}.

§4.9 Anti-derivatives, C4.9-Anti-Deriv (10 problems)

- Find anti-derivatives
- Formulation in terms of recovering position from velocity
- Exam problems:
  * Find anti-derivatives
  * Finding anti-derivatives disguised as recovering position from velocity or recovering velocity from acceleration

§5.1 S 5.1 Approximating and computing area, C5.1-Area (15 problems)

- Definition of midpoint, left and right sums
- Sigma notation, distributive rule for sums
- Sums of powers of integers
- Exam questions:
  * Estimating area with left or right sums (exclude mid-point)
  * Manipulating sums, including using formulae for sums

Part D. 6 + 3 review

§5.2 The definite integral, D5.2-Integral

- Defining area as a limit of Riemann sums
- The integral as signed area
- Properties of the integral
- Exam questions:
  - Applying linearity of the integral
  - Using area to evaluate integrals when the region under the graph is a triangle or part of a circle.

§5.3 The fundamental theorem of Calculus, Part I (evaluating integrals)

- Statement of Fundamental theorem I
- Evaluating simple integrals.

§5.4 The fundamental theorem of Calculus, Part II, D5.3-5.4-FTC (19 problems)

§5.5 The net change theorem (interpretation of FTC), D5.5-Rates (10 points)

- Exam questions
  - State fundamental theorems I and II
  - Use fundamental theorem to evaluate integrals
  - Differentiating an integral
  - Provide examples to illustrate hypotheses
  - Recovering functions from rates of change: interpretation of the fundamental theorem in applications

§5.6 Substitution method, D5.6-Substitution (10 problems) S 5.7, Further transcendental functions D5.7-Transcendental-Fun (10 problems)

- Exam questions
  - Evaluate definite and indefinite integrals by substitution
  - Be able to change limits in definite integrals
  - Anti-derivatives of $1/x$, $1/(1+x^2)$, $1/sqrt(1-x^2)$ and $1/(|x|\sqrt{x^2-1})$.

§5.8 Exponential growth and decay, D5.8-Exp-Gro-Decay (10 problems)

- Give all solutions to $y' = ky$
- Formulating simple differential equations
- Applications of exponential growth and decay including population growth and radioactive decay
- Exam questions
  - Finding functions of the form $f(t) = Ae^{kt}$ given appropriate information
  - Formulate simple differential equations

§6.1 Area of regions in the plane, D6.1-Area (10 problems)

- Writing areas as an integral
- Evaluating the integral to find the area
- Finding areas with roles of x and y interchanged
- Exam questions:
  - Expressing area between curves as an integral, including problems where the roles of x and y are reversed.
  - Evaluating area