MA 113 Calculus I, Spring 2014

Written Assignment #6

Due **Wednesday, 2 April**, at the beginning of lecture.

Instructions: The purpose of this assignment is to develop your ability to formulate and communicate mathematical arguments. Your complete assignment should have your name and section number on each page, be stapled, and be neat and legible. *Unreadable work will receive no credit.*

You should provide well-written, complete answers to each of the questions. We will look for correct mathematical arguments, careful explanations, and correct use of English. Your solution should be formulated in complete sentences. As appropriate, you may want to include diagrams or equations written out on a separate line. Your textbook provides examples of how we communicate mathematics.

Students are encouraged to use word-processing software to produce high quality solutions. However, you may find that it is simpler to add graphs and equations using pen or pencil.

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1. (4 points) Find all real numbers b and c so that the quadratic polynomial $f(x) = x^2 + bx + c$ takes on both positive and negative values. Give one example of such a pair of real numbers (b, c). Under the condition that the function takes on both positive and negative values, show that the absolute minimum of the function occurs at the midpoint between the two roots.

2. Let $C(x)$ be the cost of producing x units of a certain good with $C(0) = 0$. Assume that the graph of $C(x)$ on $[0, \infty)$ is concave up.

   (a) (4 points) Give a detailed argument showing that the average cost of producing x units, defined by $A(x) = \frac{C(x)}{x}$, is minimized at production level x_0, where x_0 is the production level for which the average cost $A(x)$ equals the estimate of the marginal cost of production given by the derivative of $C(x)$ (see page 152 in Rogawski).

   (b) (2 points) Show that the line through the points $(0, 0)$ and $(x_0, C(x_0))$ is tangent to the graph of $C(x)$.