Worksheet # 6.5: Snow Day Supplement

1. Determine if the following limits exist and evaluate each limit that exists. Carefully explain each step.

   (a) $\lim_{x\to 9} \dfrac{x-9}{\sqrt{x}-3}$

   (b) $\lim_{x\to 2} \dfrac{x+2}{x-2}$

   (c) $\lim_{s\to 5} \dfrac{1}{s-5}(\dfrac{1}{s^2} - \dfrac{1}{25})$.

2. A particle is thrown up from the ground and after t seconds its height in meters is given by the function $h(t) = -5t^2 + 20t$.

   (a) At which time(s) t is the particle on the ground?

   (b) Find the average velocity on the interval $[2, 2+s]$.

   (c) Take the limit as s approaches 0 of the expression you found in part b) to determine the instantaneous velocity at time $t = 2$.

   (d) At time $t = 2$ is the particle moving up or down?

3. Suppose that $f(x) = 1/(x-1)$.

   (a) Find the slope of the secant line through $(2, f(2))$ and $(x, f(x))$.

   (b) Take the limit as x approaches 2 of the expression in part a).

   (c) Find the equation of the tangent line to the graph of f at the point $(2, f(2))$.

4. A particle moves along a line and its position after time t seconds is $p(t) = 3t^3 + 2t$ meters to the right of the origin. Find the instantaneous velocity of the particle at $t = 2$.

5. Give the precise statement of the definition of continuity at a point.

6. Referring to the definition of continuity, explain whether or not the functions below are continuous at 2.

   (a) $f(x) = \begin{cases} \frac{x-2}{x^2-4}, & x \neq 2 \\ 4, & x = 2 \end{cases}$

   (b) $f(x) = \begin{cases} \frac{x+2}{x^2-4}, & x \neq 2 \\ 4, & x = 2 \end{cases}$

   (c) $f(x) = \begin{cases} \frac{x-2}{x^2-4}, & x \neq 2 \\ 1/4, & x = 2 \end{cases}$

7. Suppose that $0 \leq f(x) \leq x^2 + 2x + b$. There is one value of b for which we can use the squeeze theorem to find a limit of f. Find the value of b. For this value of b, give the value of a for which we can find $\lim_{x\to a} f(x)$.

8. Recall the Pythagorean identity for sin and cos, $\sin^2(x) + \cos^2(x) = 1$. Divide each term by $\cos^2(x)$ and obtain an identity involving $\tan(x)$ and $\sec(x)$.

   What identity do you obtain if you divide by $\sin^2(x)$?