MA 113 Calculus I, Spring 2015
Written Assignment #1
Due Friday, 23 January 2015 at the beginning of lecture.

Instructions: The purpose of this assignment is to develop your ability to formulate and communicate mathematical arguments. Your complete assignment should have your name and section number on each page, be stapled, and be neat and legible. *Unreadable work will receive no credit.*

You should provide well-written, complete answers to each of the questions. We will look for correct mathematical arguments, careful explanations, and correct use of English. Your solution should be formulated in complete sentences. As appropriate, you may want to include diagrams or equations written out on a separate line. Your textbook provides examples of how we communicate mathematics.

Students are encouraged to use word-processing software to produce high quality solutions. However, you may find that it is simpler to add graphs and equations using pen or pencil.

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1. (5 points) We have a sheet of cardboard that is 20 centimeters by 30 centimeters and form a box with an open top by removing squares from each of the four corners of the rectangle and folding up the flaps on each side.

   (a) Make a sketch of the sheet of cardboard showing the four corners to be removed.

   Solution: (Sketch omitted) Grading: One point. Diagram should show the squares to be removed and the lengths of the sides of the rectangle.

   (b) Write a function V that gives the volume of the resulting box in terms of x, the sidelength of the square that is removed.

   Solution: The volume of a box (or rectangular parallelepiped) is $V = \ell w h$ where ℓ, w, and h are the lengths of the three sides. When we remove the squares with sidelength x, we remove $2x$ from each of the sides of the rectangle. Thus, the base of the box will have sides of length $\ell = 30 - 2x$ centimeters and $w = 20 - 2x$ centimeters. The flaps that we fold up give the box the height x. The total volume will be

   $$V(x) = (30 - 2x)(20 - 2x)x = (600x - 100x^2 + 4x^3)\text{centimeters}^3$$

   Grading: Two points for correct formula for volume, award one of these points if at least one side of the box is computed correctly. One point for a correct explanation. Add missing units, but do not deduct for units.

   (c) Give the domain of the function V and explain why you chose this domain.

   Solution: The side length of a square should be positive. The lengths of the two squares removed is $2x$ and this must be less than the length of the smallest side. This implies that $2x < 20$ or $x < 10$. Thus, our formula will give the volume of the box for $0 < x < 10$. Thus we take the domain of V to be the open interval $(0, 10)$.

   Grading: One point for the domain with explanation. Give credit to students who give the closed interval $[0, 10]$ or who give the natural domain of the function V, $\mathbf{R}$, but point out that there is a better answer.

2. (5 points) Let f be the function defined by $f(x) = x^2 + 4x + 1$ and domain all real numbers.

(a) Solve the equation $f(x) = 1$ and explain whether or not the function f is one to one.

SOLUTION: The equation $4x^2 + 4x + 1 = 1$ is equivalent to the equation $x^2 + 4x = 0$. Factoring $x^2 + 4x = x(x + 4)$, it is easy to see that the solutions to this equation are $x = 0$ and $x = -4$. By definition a one to one function is a function so that when we fix x, we can find at most one value of x which solves the equation $f(x) = c$. Since the equation $f(x) = 1$ has two solutions, the function f is not one to one.

Grading: One point for correct answer with an explanation.

(b) Let g be the function $g(x) = x^2 + 4x + 1$ with the domain $(-\infty, a]$. Find the largest value of a for which the function g is one to one. Fix a to have this value and find the inverse function g^{-1}.

SOLUTION: If we complete the square, we may write $x^2 + 4x + 1 = x^2 + 4x + 4 - 3 = (x + 2)^2 - 3$. Since $b^2 = (-b)^2$, if the domain contains $b - 2$ and $-b - 2$ for some value $b \neq 0$, then f will not be one to one. Thus, the largest interval for which f is one to one is $(-\infty, -2]$.

To find the inverse function we must solve $g(x) = y$. This is easy to do if we complete the square and write this equation as $(x + 2)^2 - 3 = y$. Solving gives $x = -2 \pm \sqrt{y + 3}$. The range of the inverse function must be the domain of g, thus we choose the negative sign to make the range $(-\infty, -2]$.

Grading: One point for domain with explanation. Some students may argue from a graph. One point for finding the inverse function with an explanation of the choice of sign.

(c) Sketch the graphs of g and g^{-1} on the same axes and describe the relation between the two graphs.

SOLUTION: See Figure 1 for the graph. The graph of g^{-1} may be obtained from the graph of g by reflecting in the line $y = x$. One point for the graph and one point for describing the relation between the graphs of the two functions.

Deductions: Deduct one point if the paper is not written in complete sentences.

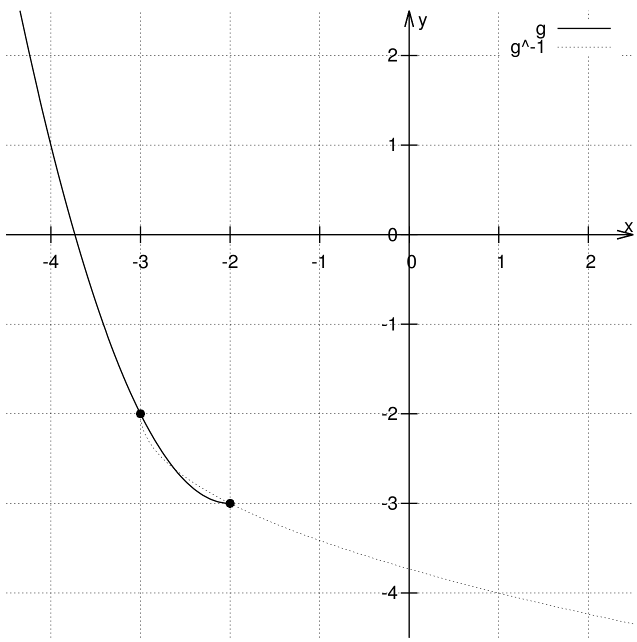

Figure 1: Graph of g and g^{-1}