MA 113 CALCULUS I, SPRING 2015
WRITTEN ASSIGNMENT #2
Due Friday, January 30, 2015, at beginning of lecture

**Instructions:** The purpose of this assignment is to develop your ability to formulate and communicate mathematical arguments. Your complete assignment should have your name and section number on each page, be stapled, and be neat and legible. *Unreadable work will receive no credit.*

You should provide well-written, complete answers to each of the questions. We will look for correct mathematical arguments, complete explanations, and correct use of English. Your solution should be formulated in complete sentences. As appropriate, you may want to include diagrams or equations written out on a separate line. You may read your textbook to find examples of how we communicate mathematics.

Students are encouraged to use word-processing software to produce high quality solutions. However, you may find that it is simpler to add graphs and equations using pen or pencil.

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1. (7 points) A soccer ball is thrown vertically up in the air starting at ground level. The height of the ball in meters after $t$ seconds is given by the equation

$$h(t) = 22.8t - 4.9t^2.$$

(a) Find the average velocity of the soccer ball over the time interval $[1, 3]$.

**Solution** The average velocity of the ball over the time interval $[1, 3]$ is given by

$$\frac{h(3) - h(1)}{3 - 1} = (22.8(3) - 4.9(3^2)) - (22.8(1) - 4.9(1^2)) = 3.2.$$

The average velocity of the ball over the time interval $[1, 3]$ is 3.2 meters per second.

(b) Compute the average velocity of the soccer ball over the time interval $[1, t]$ (Hint: Compute $h(t) - h(1)$ and show that you can factor out $(t - 1)$).

**Solution** The average velocity of the ball over the time interval $[1, t]$ is given by $\frac{h(t)-h(1)}{t-1}$. We have

$$\begin{aligned} h(t) - h(1) &= (22.8t - 4.9t^2) - (22.8(1) - 4.9(1^2)) = 22.8(t - 1) - 4.9(t^2 - 1) \\ &= (t - 1)(22.8 - 4.9(t + 1)) = (t - 1)(17.9 - 4.9t) \end{aligned}$$

Therefore, the average velocity of the ball over the interval $[1, t]$ is

$$\frac{(t - 1)(17.9 - 4.9t)}{t - 1} = 17.9 - 4.9t.$$

(c) Use the equation for the average velocity over $[1, t]$ to find the average velocity over several intervals $[1, t]$ with $t$ close to 1. Then estimate the instantaneous velocity of the soccer ball at time $t = 1$.

**Solution** For example, if $t = 1.01$, then the average velocity of the ball over the interval $[1, 1.01]$ is $17.9 - 4.9(1.01) = 12.951$. We estimate the instantaneous velocity of the ball at $t = 1$ as 13 meters per second.

(d) Sketch a graph of $h(t)$. Explain using the graph when the instantaneous velocity of the soccer ball is positive, negative, and when it is equal to 0.

**Solution** The instantaneous velocity of the ball at a given time $t$ is given by the slope of the tangent line to the graph of $y = h(t)$ at the point $(t, h(t))$. Thus the instantaneous velocity of the ball is positive, negative, or zero, when the slope of the corresponding tangent line is positive, negative, or zero. The graph of $y = h(t)$ is a parabola opening downward. The two roots of $h(t)$ are given by $t = 0$ and $t = \frac{22.8}{4.9} \approx 4.653$. The maximum value of $y = h(t)$ occurs at $\frac{22.8}{2(4.9)} \approx 2.326$. The slopes of tangent lines are positive for $t < \frac{22.8}{2(4.9)}$ and the slopes of tangent lines are negative for $t > \frac{22.8}{2(4.9)}$. The slope is zero for $t = \frac{22.8}{2(4.9)} \approx 2.326$. Note that the problem has physical meaning only when $h(t) \geq 0$, and this occurs only for $0 \leq t \leq \frac{22.8}{4.9}$.

2. (3 points) Let $f(x) = 2x^2 - x - 6$ and $g(x) = \frac{1}{x-2}$. Determine $\lim_{x\to 2} f(x)$ and $\lim_{x\to 2} g(x)$ graphically. Next, investigate $\lim_{x\to 2} f(x)g(x)$ numerically. In this example, is it true that

$$\lim_{x\to 2} f(x)g(x) = \lim_{x\to 2} f(x) \lim_{x\to 2} g(x)?$$

**Solution** $\lim_{x\to 2} f(x) = 0$ because $f(x)$ is continuous at $x = 2$ and $f(2) = 0$. The graph of $g(x)$ has a vertical asymptotic line at $x = 2$. The graph shows that $\lim_{x\to 2^+} g(x) = \infty$ and $\lim_{x\to 2^-} g(x) = -\infty$. Thus $\lim_{x\to 2} g(x)$ does not exist. Evaluating $f(x)g(x)$ for many different values of $x$ close to 2 indicates that $\lim_{x\to 2} f(x)g(x) = 7$. This is not surprising because $f(x)g(x) = \frac{(x-2)(2x+3)}{x-2} = 2x + 3$ and $2 \cdot 2 + 3 = 7$.