MA 113 Calculus I, Spring 2017

Written Assignment #3

Due Friday, February 17, 2017, at the beginning of lecture

Instructions: The purpose of this assignment is to develop your ability to formulate and communicate mathematical arguments. Your complete assignment should have your name and section number on each page, be stapled, and be neat and legible. *Unreadable work will receive no credit.*

You should provide well-written, complete answers to each of the questions. We will look for correct mathematical arguments, complete explanations, and correct use of English. Your solution should be formulated in complete sentences. As appropriate, you may want to include diagrams or equations written out on a separate line. You may read your textbook to find examples of how we communicate mathematics.

Students are encouraged to use word-processing software to produce high quality solutions. However, you may find that it is simpler to add graphs and equations using pen or pencil.

In this assignment, we will investigate the relationship between coefficients of polynomials and higher derivatives.

1. Let $f(x) = 3x^2 + 4x + 1$.
   (a) Show that $f(0) = 1$.
   (b) Show that $f'(0) = 4$.
   (c) Show that $f''(0) = 3 \cdot 2 = 6$.
2. Next, let $f(x) = a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$.
   (a) Explain why $f'''(0) = 3 \cdot 2 \cdot a_3$.
   (b) Explain why $f^{(5)}(0) = 5 \cdot 4 \cdot 3 \cdot 2 \cdot a_5$.
3. Given an arbitrary polynomial $f(x) = a_nx^n + \cdots + a_2x^2 + a_1x + a_0$, find a formula that relates the coefficient a_k to the derivative $f^{(k)}(0)$. Explain why your formula is correct.
4. Suppose that a polynomial $f(x)$ has the following values for its higher derivatives: $f(0) = 3$, $f'(0) = 3$, $f''(0) = -1$, $f'''(0) = 7$, $f^{(4)}(0) = 5$, and $f^{(n)}(0) = 0$ for all $n \geq 6$. Use your formula from the previous problem to find $f(x)$, and explain your work.
5. Is it possible for a polynomial $f(x)$ to have the following values for its higher derivatives: $f^{(n)}(0) = 1$ for all $n \geq 0$. Why or why not? **Explain your answer using the relationship between coefficients and derivatives that you have found on this assignment.**