Exam 1

Name: _____   Section: _____

    Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. You are allowed to use notes on a single piece of 8.5"X11" paper, front and back, including formulas and theorems. **You are required to turn this page in with your exam.** You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS). Absolutely no communication device use during the exam is allowed.

    The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

    Show <u>all work</u> to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ   **6** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

2 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ   **7** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

3 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ   **8** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

4 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ   **9** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

5 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ   **10** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

Trig Identities

- $\sin^2(x) + \cos^2(x) = 1$ and $\tan^2(x) + 1 = \sec^2(x)$
- $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$ and $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$

Multiple Choice Questions

1. (5 points) Find $\displaystyle\int x^2 e^x \, dx$.

   A. $e^{x^2} 2x + C$.

   B. $xe^{x^2} + 2x + C$.

   C. $x^2 e^x - 2xe^x + 2e^x + C$.

   D. $\frac{1}{3}x^3 e^x + C$.

   E. $x^2 e^x + 2e^x + C$.

2. (5 points) If $f(0) = 2$, $f(1) = 2$, $f'(0) = 1$ and $f'(1) = 3$, and $f''(x)$ is continuous, what is $\displaystyle\int_0^1 (x+1)f''(x) \, dx$?

   A. 5

   B. -1

   C. 6

   D. -4

   E. 1

3. (5 points) Find $\int \sin^2(3x)\, dx$.

    A. $\frac{1}{3}\sin^3(3x)\cos(3x) + C$.

    B. $6\sin(3x) + C$.

    C. $-\frac{1}{3}\cos^2(3x) + C$.

    D. $\frac{1}{2}x - \frac{1}{12}\sin(6x) + C$.

    E. $-\frac{1}{3}\sin^3(3x)\cos(3x) + C$.

4. (5 points) Which of the following is equal to the integral

$$\int \left(\frac{1}{\sqrt{16 - x^2}}\right)^3 dx$$

after making the substitution $x = 4\sin(\theta)$?

    A. $-\dfrac{1}{16}\int \csc^2(\theta)\, d\theta$.

    B. $\dfrac{1}{16}\int \sec^2(\theta)\, d\theta$.

    C. $\dfrac{1}{4}\int \sec^2(\theta)\, d\theta$.

    D. $-\dfrac{1}{4}\int \csc^2(\theta)\, d\theta$.

    E. $\dfrac{1}{16}\int \cos^2(\theta)\, d\theta$.

5. (5 points) Find the limit:
$$\lim_{n\to\infty} \frac{2n^2 + 5}{n^2 + 7}.$$

    A. 1

    B. 2

    C. -2

    D. -4

    E. 8

6. (5 points) Find $\displaystyle\int_1^\infty \frac{1}{x^{\frac{8}{3}}}\, dx$

    A. $\frac{8}{3}$

    B. $\frac{5}{3}$

    C. $\frac{2}{3}$

    D. $\frac{3}{8}$

    E. $\frac{3}{5}$

7. (5 points) What is the form of the partial fraction decomposition of

$$\frac{x^2 - 2}{(x+1)^2(x^2+2)(x-1)}?$$

    A. $\dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+2} + \dfrac{Dx+E}{x^2-1}$

    B. $\dfrac{A}{x+1} + \dfrac{B}{(x+1)^2} + \dfrac{Dx+E}{x^2+2}$

    C. $\dfrac{A}{x+1} + \dfrac{B}{(x+1)^2} + \dfrac{C}{x-1} + \dfrac{Dx+E}{x^2+2}$

    D. $\dfrac{A}{x+1} + \dfrac{Bx+C}{x^2-1}$

    E. $\dfrac{A}{x+1} + \dfrac{B}{(x+1)^2} + \dfrac{C}{x-1}$

8. (5 points) Find the coefficient B in the partial fraction decomposition

$$\frac{x}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

    A. $B = \dfrac{1}{2}$

    B. $B = -1$

    C. $B = 1$

    D. $B = -\dfrac{1}{2}$

    E. $B = -2$

9. (5 points) Let $f(x)$ be a function that satisfies $|f''(x)| \le 3$ on the interval $[3, 5]$. Choose the smallest n so that we can be sure that $E_M = |M_n - \int_3^5 f(x)dx| \le .0001$, where M_n is the midpoint approximation with n intervals.

    A. $n = 100$

    B. $n = 500$

    C. $n = 10,000$

    D. $n = 200$

    E. $n = 50$

10. (5 points) Find the Simpson's rule estimate of $\int_1^5 x^3 \, dx$ for $n = 4$.

    A. $S_4 = \dfrac{424}{3}$

    B. $S_4 = \dfrac{324}{3}$

    C. $S_4 = 162$

    D. $S_4 = 78$

    E. $S_4 = 156$

Free Response Questions

11. (a) (2 points) Compute $\int x\cos(x)\ dx$

(b) (8 points) Compute $\int_0^{\frac{\pi}{2}} x\cos(x)\ dx$

12. (10 points) Compute $\int \sqrt{4 - x^2} \, dx$. You must simplify your answer.

13. (10 points) Compute $\displaystyle\int_1^\infty x^2 e^{-x^3}\, dx$.

14. (10 points) Using the method of partial fractions, compute

$$\int \frac{x+1}{(x+2)(x^2+1)}\, dx.$$

15. (a) (5 points) Use the midpoint rule to estimate the integral

$$\int_1^9 \frac{1}{x^2}\, dx$$

Use four intervals (ie find M_4).

(b) (5 points) Use the trapezoid rule to estimate the integral

$$\int_1^9 \frac{1}{x^2} dx$$

Use four intervals (ie find T_4).