Calculus II   Russell Brown
Exam 1   8 February 1900

Answer all of the following questions. Use the answer sheets provided.
Additional sheets are available if necessary. No books or notes may be used.
You may use a calculator. When answering these questions, please be sure
to 1) check answers when possible, 2) clearly indicate your answer and the
reasoning used to arrive at that answer *(unsupported answers may receive
NO credit)*.

Name _____

Section _____

Question	Score	Total
1		10
2		15
3		15
4		15
5		15
6		10
7		10
8 or 9		10
Total		100

1. Find the length of the curve $x(t) = t^2$, $y(t) = t^3$ for $1 \le t \le 3$.

2. Consider a tank whose vertical cross section is a right triangle as shown and the length of the tank is 8 feet. The tank is full of water whose (weight) density is 62.4 lbs/ft^3.

(a) Let $y = 0$ correspond to the bottom of the tank and consider the volume of water between y and $y + \Delta y$. Approximate this volume by a box and give the volume of this box.

(b) Give the work needed to lift the volume of water from your answer in part a to a height 3 feet above the top of the tank.

(c) Write down an integral which gives the total work needed to empty the tank through an outlet which is located 3 feet above the top of the tank.

(d) Evaluate the integral from part c). What are the units for this answer.

3. Suppose $f(x) = \frac{1}{2}x^3 + x + 1$.

    (a) Sketch $f(x)$.

    (b) Sketch $f^{-1}(x)$.

    (c) Find $f^{-1}(\frac{5}{2})$. Hint: This is easily done by inspection.

    (d) Find $f^{-1\prime}(\frac{5}{2})$.

4. Compute the following derivatives.

(a) $D_x x^2 e^x$.

(b) $D_x \dfrac{e^x + 1}{e^x - 1}$

(c) $D_x \ln(x^{2x})$

(d) $D_x \ln(\cos x)$

(e) $D_x \tan^{-1}(2x)$

5. Compute the following integrals.

(a) $\displaystyle\int_2^3 \frac{x+1}{x}\,dx$

(b) $\displaystyle\int \frac{e^x}{e^x+1}\,dx$

(c) $\displaystyle\int_0^2 \frac{x}{1+x^4}\,dx$

6. Solve the initial value problem:

$$y' - xy = 0 \qquad y(0) = 2.$$

7. Give exact values for the following. You may use your calculator to check your answer, but you must explain the reasoning for computing these quantities without a calculator.

   (a) $\sin(\sin^{-1}(1/4))$

   (b) $\sin(\cos^{-1}(-1/3))$

   (c) $\cos(2\sin^{-1}(1/5))$

Do exactly one of the following problems. Indicate clearly which
problem is to be graded.

8. (a) Give the definition of $\ln x$.

    (b) If $a > 0$ and $b > 0$, prove that

    $$\ln(ab) = \ln a + \ln b.$$

9. Let $f(x)$ be the inverse function for $\cos x$ on the interval $[0, \pi]$.

    (a) Differentiate the equation

    $$\cos(f(x)) = x$$

    and express $f'(x)$ in terms of f and a trigonometric function.

    (b) Use trigonometric identities to simplify your answer from part a)
    and express $f'(x)$ only in terms of x.