MA 114 — Calculus II Spring 2014
Exam 4 Feb. 11, 2014

Name: ____________________________

Section: ____________________________

Last 4 digits of student ID #: ________

- No books or notes may be used.
- Turn off all your electronic devices and do not wear ear-plugs during the exam.
- You may use a calculator, but not one which has symbolic manipulation capabilities or a QWERTY keyboard.
- Additional blank sheets for scratch work are available upon request.
- **Multiple Choice Questions:**
  Record your answers on the right of this cover page by marking the box corresponding to the correct answer.
- **Free Response Questions:**
  Show all your work on the page of the problem. Clearly indicate your answer and the reasoning used to arrive at that answer.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		20
5		13
6		13
7		18
8		18
9		18
Total		100

Unsupported answers for the free response questions may not receive credit!

Record the correct answer to the following problems on the front page of this exam.

1. Compute the average value of the function $f(x) = 3x^2 + 2$ over the interval $[1, 4]$.

A. 14

B. 17

C. 20

D. 23

E. 26

$$\frac{1}{4-1}\int_1^4 (3x^2+2)\,dx = \frac{1}{3}\left(x^3+2x\right)\Big|_1^4$$

$$= \frac{1}{3}(64+8-1-2) = \underline{23}$$

2. Which trigonometric substitution is needed to evaluate the integral

$$\int \frac{1}{\sqrt{x^2+10}}\,dx\,?$$

A. $x = 10\sin(\theta)$.

B. $x = \sqrt{10}\sec(\theta)$.

C. $x = \sqrt{10}\tan(\theta)$.

D. $x = 10\sec(\theta)$.

E. $x = \sqrt{10}\sin(\theta)$.

Record the correct answer to the following problems on the front page of this exam.

3. Consider the region in the first quadrant enclosed by the graphs of

$$f(x) = 2 - x^2, \quad g(x) = x, \quad \text{and } x = 0.$$

When rotating this region about the y-axis, which of the following integrals gives the volume of the resulting solid of revolution?

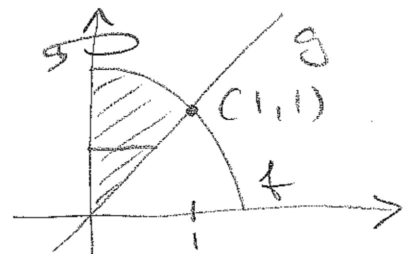
A. $\pi \int_{-\sqrt{2}}^{\sqrt{2}} (2 - x^2)^2 dx$

B. $2\pi \int_{0}^{\sqrt{2}} (2 - x^2)^2 dx$

C. $\pi \int_{0}^{1} \left((2 - x^2)^2 - x^2\right) dx$

D. $2\pi \int_{0}^{1} (2 - x^2 - x) dx$

E. $2\pi \int_{0}^{1} x(2 - x^2 - x) dx$



Shell Method:

$V = 2\pi \int_0^1 x(f(x) - g(x))\, dx$

4. Let $f(x) = \sqrt{4x + 6}$ and $g(x) = 3$. The region enclosed by the graphs of f and g over the interval $[1, 3]$ is rotated about the x-axis. Which of the following integrals expresses the volume of the resulting solid of revolution?

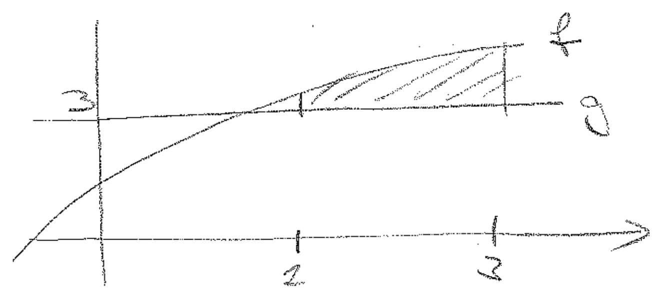
A. $2\pi \int_{1}^{3} x\left(\sqrt{4x + 6} - 3\right) dx.$

B. $\pi \int_{1}^{3} (\sqrt{4x + 6} - 3)^2 dx.$

C. $\pi \int_{1}^{3} (4x - 3) dx.$

D. $\pi \int_{1}^{3} x\left((4x + 6)^2 - 9\right) dx.$

E. $2\pi \int_{1}^{3} x(4x - 3) dx.$



Washer Method:

$V = \pi \int_1^3 (f^2(x) - g^2(x))\, dx$

5. Evaluate the integral

$$\int x^3 \ln(2x)dx.$$

$$\int \underbrace{x^3}_{v'} \underbrace{\ln(2x)}_{u} dx \quad \overline{\quad u' = \frac{1}{x} \quad} \quad \frac{1}{4}x^4 \ln(2x) - \frac{1}{4}\int \frac{1}{x} x^4 dx$$

$$v = \frac{1}{4}x^4$$

$$= \frac{1}{4}\left(x^4 \cdot \ln(2x) - \int x^3 dx\right)$$

$$= \frac{1}{4}\left(x^4 \cdot \ln(2x) - \frac{1}{4}x^4\right) + C$$

Free Response Questions: Show your work!

6. Compute the integral

$$\int \sin^2 x \cos^5 x dx.$$

$$= \int \sin^2 x \overbrace{(1-\sin^2 x)^2}^{\cos^4 x} \cos x \, dx$$

$$= \int u^2 (1-u^2)^2 du = \int u^2 - 2u^4 + u^6 du$$

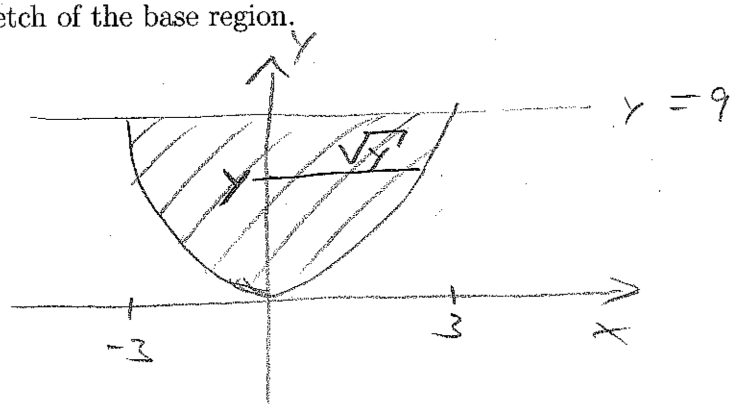
$u = \sin x$
$du = \cos x \, dx$

$$= \frac{1}{3} u^3 - \frac{2}{5} u^5 + \frac{1}{7} u^7 + C$$

$$= \frac{1}{3} \sin^3 x - \frac{2}{5} \sin^5 x + \frac{1}{7} \sin^7 x + C$$

7. Consider the solid whose base is the region enclosed by the graphs of $y = 9$ and $y = x^2$ and whose vertical cross sections perpendicular to the y-axis at the value y are rectangles of height $5y$.

(a) Give a sketch of the base region.



(b) Compute the area of the cross section at y.

$A(y) = \text{width} \cdot \text{height} = 2\sqrt{y} \cdot 5y$

$= 10y^{3/2}$

(c) Compute the volume of the solid.

$$V = \int_0^9 10y^{3/2}\,dy = 10 \cdot \frac{2}{5} y^{5/2} \Big|_0^9$$

$$= 4(3^5 - 0) = 4 \cdot 243 = \underline{972}$$

Free Response Questions: Show your work!

8. Evaluate the integral $\int \frac{dx}{x^2\sqrt{x^2-16}}$.

$$\overline{}$$

$$x = 4\sec\theta$$

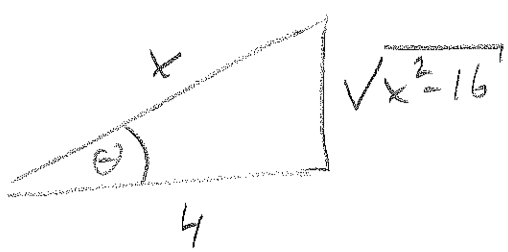
$$dx = 4\sec\theta\tan\theta\, d\theta$$

$$\sqrt{x^2-16} = 4\tan\theta$$

$$\int \frac{4\sec\theta\cancel{\tan\theta}\,d\theta}{16\sec^2\theta\cdot 4\cancel{\tan\theta}}$$

$$= \frac{1}{16}\int \frac{1}{\sec\theta}\,d\theta = \frac{1}{16}\int \cos\theta\, d\theta$$

$$= \frac{1}{16}\sin\theta + C = \underline{\frac{1}{16}\frac{\sqrt{x^2-16}}{x} + C}$$

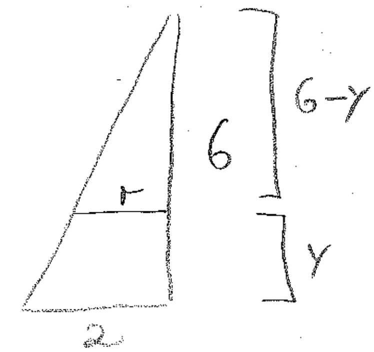


9. A cone with a circular base of radius 2 m and height 6 m is to be built with material having a density of 180 kg per m^3.

(a) Compute the area of the cone's cross section at height y above the base.

Similar triangles:

$$\frac{6}{2} = \frac{6-y}{r}, \text{ so } r = 2 - \frac{y}{3}$$



$A(y) = \pi\left(2 - \frac{y}{3}\right)^2$ because the cross section is a disk with radius $r = 2 - \frac{y}{3}$.

(b) Present the integral that expresses the work against gravity to build the cone. For gravity use $9.8\,m/s^2$.

$$W = \int_0^6 L(y)\,dy, \text{ where}$$

$$L(y) = \underbrace{9.8}_{\text{gravity}} \cdot \underbrace{180}_{\text{density}} \cdot \underbrace{\pi\left(2 - \frac{y}{3}\right)^2}_{\text{cross-section}} \cdot \underbrace{y}_{\text{vertical distance of work}}$$

(c) Calculate the work. Give the unit with your answer.

$$W = 9.8 \cdot \pi \int_0^6 180\left(4 - \frac{4}{3}y + \frac{y^2}{9}\right) y\,dy$$

$$= 20 \cdot 9.8\pi \int_0^6 (36y - 12y^2 + y^3)\,dy$$

$$= 20 \cdot 9.8\pi \left(18y^2 - 4y^3 + \frac{1}{4}y^4\right)\Big|_0^6$$

$$= 20 \cdot 9.8\pi \cdot 36(18 - 24 + 9)$$

$$= 60 \cdot 9.8 \cdot \pi \cdot 36 = \underline{21{,}168\pi \text{ N-m}}$$