Name: ______________________________ Section: ______________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. No books or notes may be used. You may use a graphing calculator during the exam, but you may not use a calculator that has symbolic manipulation capabilities of any sort. This forbids the use of TI-89, TI-Nspire CAS, HP 48, TI 92, and many others, as stated on the syllabus. Absolutely no cell phone use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. You should also show your work on the multiple choice questions as it will make it easier for you to check your work. You should give exact answers, rather than a decimal approximation unless the problem asks for a decimal answer. Thus, if the answer is 2π, you should not give a decimal approximation such as 6.283 as your final answer.

Multiple Choice Questions

1 A B C D E **6** A B C D E

2 A B C D E **7** A B C D E

3 A B C D E **8** A B C D E

4 A B C D E **9** A B C D E

5 A B C D E **10** A B C D E

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

Multiple Choice Questions

1. (5 points) Select the answer that is equal to $\int e^{x/2} f(x)\, dx$.

   A. $2e^{x/2} f(x) + 2 \int e^{x/2} f'(x)\, dx$

   B. $e^{x/2} f(x) - \int e^{x/2} f'(x)\, dx$

   C. $2e^{x/2} f(x) - 2 \int e^{x/2} f'(x)\, dx$

   D. $e^{x/2} f(x) - 2 \int e^{x/2} f'(x)\, dx$

   E. $\frac{1}{2} e^{x/2} f(x) - \frac{1}{2} \int e^{x/2} f'(x)\, dx$

2. (5 points) What substitution would be most useful to evaluate the integral $\int \sqrt{16 - x^2}\, dx$?

   A. $u = 4\sin(x)$

   B. $x = 4\tan(\theta)$

   C. $u = 2\sin(x)$

   D. $2x = \sin(\theta)$

   E. $x = 4\sin(\theta)$

3. (5 points) Four of the options below might appear in the partial fractions decomposition of

$$f(x) = \frac{1}{(x^2+x+1)(x^2-1)(x+1)}.$$

Select the option that does NOT appear in the partial fractions decomposition of f.

A. $\dfrac{A}{x+1}$

B. $\dfrac{B}{(x+1)^2}$

C. $\dfrac{C}{x-1}$

D. $\dfrac{D}{(x-1)^2}$

E. $\dfrac{Ex+F}{x^2+x+1}$

4. (5 points) Let L_n, R_n, and T_n be left endpoint, right endpoint, and trapezoidal approximations to the integral $I = \int_0^2 (9-x^2)\,dx$. Which of the following is true?

A. $R_n < T_n < I < L_n$

B. $T_n < I < R_n < L_n$

C. $R_n < L_n < T_n < I$

D. $L_n < I < R_n < T_n$

E. $R_n < I < T_n < L_n$

5. (5 points) Find the value of A in the partial fractions decomposition

$$\frac{2x-1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}.$$

A. $A = 1$

B. $A = 3$

C. $A = 2$

D. $A = -3$

E. $A = -1$

6. (5 points) Find the antiderivative $\int 15\sin^2(x)\cos^3(x)\,dx$.

   A. $5\cos^3(x) - 3\sin^5(x) + C$

   B. $5\sin^3(x) - 3\sin^5(x) + C$

   C. $\cos^3(x) - \sin^5(x) + C$

   D. $5\sin^3(x) + 3\sin^5(x) + C$

   E. $3\sin^3(x) - 5\sin^5(x) + C$

7. (5 points) If $x = 2\tan(t)$ and $-\pi/2 < t < \pi/2$, find $\sin(t)$.

   A. $2\sqrt{x^2+4}$

   B. $x/\sqrt{x^2-4}$

   C. $2/\sqrt{4-x^2}$

   D. $2/\sqrt{x^2+4}$

   E. $x/\sqrt{x^2+4}$

8. (5 points) For which values of a does the integral $\int_0^\infty \frac{dx}{(x+1)^a}$ converge?

   A. $(2,\infty)$

   B. $(-\infty,1)$

   C. $(-\infty,2]$

   D. $(1,\infty)$

   E. $[1,\infty)$

9. (5 points) A car is driving east along a straight road in west Texas. The table below gives its velocity every fifteen minutes over the course of an hour.

Time (minutes)	0	15	30	45	60
Velocity (miles/hour)	60	80	85	80	75

Use the trapezoidal rule with 4 subintervals to estimate the distance travelled in the hour and select the option below that is closest.

A. 76

B. 77

C. 78

D. 79

E. 80

10. (5 points) All of the integrals below except one are improper integrals. Select the answer that is NOT an improper integral.

A. $\int_0^{\pi} \tan(x)\,dx$

B. $\int_1^3 \frac{1}{x^2+x+6}\,dx$

C. $\int_1^3 \frac{1}{x^2+x-6}\,dx$

D. $\int_1^{\infty} e^{-x}\,dx$

E. $\int_1^{\infty} x\,dx$

Free Response Questions

11. (10 points) Evaluate the integral

$$\int_1^2 \frac{\ln(x)}{x^2}\,dx.$$

Solution: Integration by parts with

$$u = \ln(x), \quad dv = \frac{1}{x^2}\,dx.$$

Thus

$$du = \frac{1}{x}\,dx, \quad v = -\frac{1}{x}.$$

Then

$$\begin{aligned}
\int_1^2 \frac{\ln(x)}{x^2}\,dx &= \left[-\frac{\ln(x)}{x}\right]_{x=1}^{x=2} + \int_1^2 \frac{1}{x^2}\,dx \\
&= \left[-\frac{\ln(x)}{x} - \frac{1}{x}\right]_{x=1}^{x=2} \\
&= \left(-\frac{\ln(2)}{2} - \frac{1}{2}\right) - (0-1) \\
&= \frac{1-\ln(2)}{2}.
\end{aligned}$$

12. (10 points) Find the antiderivative

$$\int \frac{dx}{(4-x^2)^{3/2}}.$$

Solution: Trigonometric substitution: $x = 2\sin(\theta)$ $(-\pi/2 \le \theta \le \pi/2)$. Then

$$(4-x^2)^{1/2} = \sqrt{4-x^2} = 2\cos(\theta), \quad \text{and} \quad dx = 2\cos(\theta)\, d\theta.$$

Thus

$$\begin{aligned} \int \frac{dx}{(4-x^2)^{3/2}} &= \int \frac{2\cos\theta\, d\theta}{8\cos^3(\theta)} \\ &= \frac{1}{4}\int \sec^2(\theta)\, d\theta \\ &= \frac{1}{4}\tan(\theta) + C. \end{aligned}$$

Since

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{2\sin(\theta)}{2\cos(\theta)} = \frac{x}{\sqrt{4-x^2}},$$

we have

$$\int \frac{dx}{(4-x^2)^{3/2}} = \frac{x}{4\sqrt{4-x^2}} + C.$$

13. (10 points) If we use Simpson's Rule S_n with n subintervals (n even) to approximate the integral $I = \int_a^b f(x)\,dx$, then the error $E_S = I - S_n$ satisfies the estimate

$$|E_S| \le \frac{K(b-a)^5}{180\,n^4}.$$

Here K is an upper bound for $|f^{(4)}(x)|$ and thus $|f^{(4)}(x)| \le K$ for $a \le x \le b$.
Let $f(x)$ be a function whose fourth derivative $f^{(4)}(x) = \cos(x^2)$. If we use Simpson's Rule with n subintervals to approximate

$$I = \int_0^{\pi/2} f(x)\,dx,$$

find the smallest number n for which the error $|I - S_n|$ is at most 0.0001. [Note: Recall that n must be even.]

Solution: Since $f^{(4)}(x) = \cos(x^2)$, the maximal value of $|f^{(4)}(x)|$ is $K = 1$. Thus

$$|I - S_n| \le \frac{(\pi/2)^5}{180\,n^4}.$$

It follows that $|I - S_n| \le 0.0001$ when

$$n^4 \ge \frac{(\pi/2)^5}{0.0180} \approx 531.2842,$$

i.e.,

$$n \ge 4.8010.$$

Since n must be even, we need to take $n = 6$.

14. (10 points) Find the antiderivative

$$\int \frac{(x+1)^2}{x^3+x}\,dx.$$

Solution: Using partial fractions. Since $x^3 + x = x(x^2 + 1)$, we write

$$\frac{x^2+2x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}.$$

Thus

$$\begin{aligned} x^2 + 2x + 1 &= A(x^2+1) + x(Bx+C) \\ &= (A+B)x^2 + Cx + A, \end{aligned}$$

and comparing the coefficients of x^2, x, and 1, we have

$$A + B = 1, \quad C = 2, \quad A = 1.$$

Solving for A, B, and C, gives $A = 1, B = 0, C = 2$. Thus

$$\frac{x^2+2x+1}{x(x^2+1)} = \frac{1}{x} + \frac{2}{x^2+1}$$

and

$$\int \frac{(x+1)^2}{x^3+x}\,dx = \ln|x| + 2\tan^{-1}(x) + C.$$

15. (a) (4 points) Use the definition to write the improper integral

$$\int_0^\infty \frac{dx}{(2x+1)^2}$$

as a limit of proper integrals.

Solution:

$$\int_0^\infty \frac{dx}{(2x+1)^2} = \lim_{t\to\infty} \int_0^t \frac{dx}{(2x+1)^2}.$$

(b) (6 points) Evaluate the expression you found in part (a) and determine if the improper integral is convergent. If it is convergent, give the value.

Solution: We have

$$\begin{aligned} \int_0^t \frac{dx}{(2x+1)^2} &= \left[\frac{1}{2}\left(-\frac{1}{2x+1}\right)\right]_{x=0}^{x=t} \\ &= \left(-\frac{1}{2(2t+1)}\right) - \left(-\frac{1}{2}\right) \\ &= \frac{t}{2t+1}. \end{aligned}$$

Thus the improper integral converges and

$$\int_0^\infty \frac{dx}{(2x+1)^2} = \lim_{t\to\infty} \frac{t}{2t+1} = \frac{1}{2}.$$