Exam 1

Name: _____ Section: _____

    Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. You are allowed to use notes on a single piece of 8.5"X11" paper, front and back, including formulas and theorems. **You are required to turn this page in with your exam.** You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS). Absolutely no communication device use during the exam is allowed.

    The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

    Show <u>all work</u> to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1 (A) (B) (C) (D) (E)   **6** (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)   **7** (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)   **8** (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)   **9** (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)   **10** (A) (B) (C) (D) (E)

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

Trig Identities

- $\sin^2(x) + \cos^2(x) = 1$ and $\tan^2(x) + 1 = \sec^2(x)$
- $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$ and $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$

Multiple Choice Questions

1. (5 points) Find $\displaystyle\int x\sin(x)\ dx$.

     A. $\sin(x) + \cos(x) + C$.

     B. $\dfrac{x^2}{2}(\sin(x) - \cos(x)) + C$.

     C. $\sin(x) - x\cos(x) + C$.

     D. $\cos(x^2) + \sin(x) + C$.

     E. $2x\sin(x) + C$.

2. (5 points) If $f(0) = 1$, $f(1) = 1$, $f'(0) = 2$ and $f'(1) = 7$, and $f''(x)$ is continuous, what is $\displaystyle\int_0^1 (x - 3)f''(x)\ dx$?

     A. 1

     B. 15

     C. -6

     D. 7

     E. -8

3. (5 points) Find $\int (1 + \cos(x))^2 \ dx$.

     A. $\dfrac{3}{2}x + 2\sin(x) + \dfrac{1}{4}\sin(2x) + C$.

     B. $1 + \sin^2(x) + C$.

     C. $\cos(2x) + \dfrac{3}{4}x + C$.

     D. $\cos(2x) + 2\cos(x) + x + C$.

     E. $\cos(x)^2 \sin(2x) + C$.

4. (5 points) Which of the following is equal to the integral

$$\int \frac{x^3}{\sqrt{4 - x^2}} \ dx$$

after making the substitution $x = 2\sin(\theta)$?

     A. $4 \int \sec(\theta) \ d\theta$.

     B. $4 \int \cos^2(\theta) \ d\theta$.

     C. $8 \int \sin(\theta)\cos(\theta) \ d\theta$.

     D. $8 \int \sin^3(\theta) \ d\theta$.

     E. $2 \int \sin^2(\theta)\sec(\theta) \ d\theta$.

5. (5 points) Which of the following is equal to the expression $\tan(\arcsin(\frac{x}{4}))$?

     A. $\dfrac{x}{4}$.

     B. $\dfrac{x}{\sqrt{16 - x^2}}$.

     C. $\dfrac{1}{4}\sqrt{4 + x^2}$.

     D. $\dfrac{x}{\sqrt{4 + x^2}}$.

     E. $\dfrac{16}{\sqrt{x^2 - 16}}$.

6. (5 points) Find

$$\int_1^\infty \frac{1}{x^{\frac{3}{2}}}\, dx$$

A. ∞

B. $\dfrac{3}{2}$

C. 2

D. $\dfrac{2}{3}$

E. $-\infty$

7. (5 points) What is the form of the partial fraction decomposition of

$$\frac{x-2}{(x+1)(x^2+x+1)(x-1)}?$$

A. $\dfrac{A}{x+1} + \dfrac{B}{x-1} + \dfrac{Cx+D}{x^2+x+1}$

B. $\dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+x+1} + \dfrac{Dx+E}{x-1}$

C. $\dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+x+1}$

D. $\dfrac{A}{x+1} + \dfrac{Bx+C}{x-1}$

E. $\dfrac{A}{x+1} + \dfrac{B}{x^2+x+1}$

8. (5 points) Find the coefficient B in the partial fraction decomposition

$$\frac{x}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$$

A. $B = \dfrac{1}{3}$

B. $B = 0$

C. $B = \dfrac{2}{3}$

D. $B = \dfrac{1}{2}$

E. $B = -\dfrac{1}{3}$

9. (5 points) Let $f(x)$ be a function that satisfies $|f''(x)| \le 3$ on the interval $[5, 7]$. Choose the smallest n so that we can be sure that $E_M = |M_n - \int_5^7 f(x)dx| \le .000001$, where M_n is the midpoint approximation with n intervals.

     A. $n = 15,000$

     B. $n = 1,000$

     C. $n = 500$

     D. $n = 200$

     E. $n = 100$

10. (5 points) Find the Simpson's rule estimate of $\int_1^7 x^2 \, dx$ for $n = 6$.

     A. $S_6 = 19$

     B. $S_6 = 342$

     C. $S_6 = 115$

     D. $S_6 = 114$

     E. $S_6 = 216$

Free Response Questions

11. (10 points) Compute $\displaystyle\int e^x \cos(x)\, dx$.

12. (10 points) Compute $\int \sqrt{9 + x^2} \, dx$. You must simplify your answer.

13. (10 points) Compute $\displaystyle\int_{1}^{\infty} xe^{x^2+1}\, dx$. **Justify your answer by showing your work.**

14. (10 points) Using the method of partial fractions, compute

$$\int \frac{1}{(x-3)(x^2-2x+1)}\, dx.$$

15. (a) (5 points) Use the midpoint rule to estimate the integral

$$\int_1^7 \frac{1}{x^3}\, dx$$

Use six intervals (ie find M_6).

(b) (5 points) Use the trapezoid rule to estimate the integral

$$\int_1^7 \frac{1}{x^3} dx$$

Use six intervals (ie find T_6).