

Name: _____

Section: _____

Last 4 digits of student ID #: _____

- No books or notes may be used.
- Turn off all your electronic devices and do not wear ear-plugs during the exam.
- You may use a calculator, but not one which has symbolic manipulation capabilities or a QWERTY keyboard.
- Additional blank sheets for scratch work are available upon request.
- **Multiple Choice Questions:**
Record your answers on the right of this cover page by marking the box corresponding to the correct answer.
- **Free Response Questions:**
Show all your work on the page of the problem. Clearly indicate your answer and the reasoning used to arrive at that answer.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		20
5		16
6		15
7		18
8		16
9		15
Total		100

Unsupported answers for the free response questions may not receive credit!

Record the correct answer to the following problems on the front page of this exam.

1. Let f be a differentiable function. Which of the following expressions is the same as the integral

$$\int \frac{1}{x^2} f(x) dx.$$

- A. $-\frac{f(x)}{x} + \int \frac{f'(x)}{x} dx.$
- B. $-\frac{2f(x)}{x^3} + \int \frac{2f'(x)}{x^3} dx.$
- C. $\frac{f'(x)}{x} - \int \frac{f(x)}{x} dx.$
- D. $\frac{f(x)}{x^2} - \frac{f'(x)}{x}.$
- E. $\frac{f'(x)}{x^3} - \int \frac{f(x)}{x^3} dx.$

2. A solid has a circular base with radius 3 centered at 0. Its cross sections perpendicular to the x -axis are squares. Which of the following expresses the volume of the solid?

- A. $\int_{-3}^3 (9 - x^2) dx.$
- B. $\pi \int_{-3}^3 (9 - x^2)^2 dx.$
- C. $4 \int_{-3}^3 (9 - x^2) dx.$
- D. $\pi \int_{-3}^3 \sqrt{9 - x^2} dx.$
- E. $2\pi \int_{-3}^3 x(9 - x^2) dx.$

Record the correct answer to the following problems on the front page of this exam.

3. Compute the average value of the function $f(x) = \sin(x)$ over the interval $[0, \pi]$.

A. $\frac{16}{25}$.

B. $\frac{-2}{\pi}$.

C. 2.

D. $\frac{2}{\pi}$.

E. 0.

4. Consider the region in the first quadrant bounded by the graph of $f(x) = x^2$ and the line $x = 2$. Compute the volume of the solid obtained by rotating the region about the y -axis.

A. 4π .

B. $\frac{32\pi}{5}$.

C. 8π .

D. 25.

E. $\frac{16\pi}{5}$.

Free Response Questions: Show your work!

5. (a) Let a, b be real numbers. Use calculus to compute the integral $\int (ax + b) \sin(x) dx$.

(b) Use calculus to compute the integral $\int_0^{\pi/2} \sin^2(x) \cos^3(x) dx$.

Free Response Questions: Show your work!

6. In a particular neighborhood in Lexington the density of the squirrel population is given by

$$\rho(r) = \frac{160}{1+r^2} \text{ squirrels per square kilometer,}$$

where r is the distance (in km) from the backyard of squirrel lover Dr. Nuts.

- (a) Write an integral that expresses the total population of squirrels within a 2-km radius of that backyard.

- (b) Evaluate the integral in (a). Give the exact answer.

Free Response Questions: Show your work!

7. The region bounded by

$$y = \sqrt{x}, \quad y = 1, \quad \text{and} \quad x = 0$$

is rotated about the y -axis to form a solid.

(a) Draw a CLEAR picture of the region described by the three equations.

(b) Write an integral that uses the *disk method* to compute the volume.

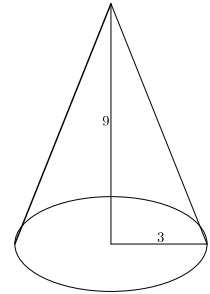
(c) Write an integral that uses the *shell method* to compute the volume.

(d) Select one of the above integrals and evaluate it to compute the volume.

Free Response Questions: Show your work!

8. A right cone with a circular base of radius 3m and height 9m is to be built with a material having a density of 30 kg per m^3 . See the figure for such a cone.

(a) Compute the area of the cross section at height y above the base. Also give the unit with your answer.



(b) Present the integral that expresses the work against gravity to build the cone. Use that the acceleration due to gravity is 9.8 m/s^2 and that all the material to be used is lying on the ground. Give the unit with your answer. **Do not evaluate the integral.**

Free Response Questions: Show your work!

9. (a) Give the Taylor series centered at 0 of the function

$$f(x) = \frac{1}{1-x}.$$

- (b) Use the result from (a) to find the Taylor series centered at 0 for

$$g(x) = \frac{1}{(1-x)^2}.$$

- (c) Find the sum of the series

$$\frac{1}{2} + 2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right)^3 + 4\left(\frac{1}{2}\right)^4 + \dots$$

and express your answer as a rational number a/b .

[Hint: Use the result from part (b) and multiply by x .]