Exam 2

Name: ______________________________ Section: ______________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. You are allowed to use notes on a single piece of 8.5" x 11" paper, front and back, including formulas and theorems. **You are required to turn this page in with your exam.** You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS). Absolutely no communication device use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **6** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

2 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **7** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

3 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **8** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

4 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **9** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

5 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **10** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

Multiple Choice Questions

1. (5 points) Give the first four terms of the sequence $\{a_1, a_2, a_3, a_4\}$ defined by

$$a_n = \frac{2n}{\sqrt{n^2+1}}.$$

A. $\{\frac{2}{\sqrt{2}}, \frac{4}{\sqrt{5}}, \frac{6}{\sqrt{10}}, \frac{8}{\sqrt{17}}\}$
B. $\{\frac{2}{2}, \frac{4}{3}, \frac{6}{4}, \frac{8}{5}\}$
C. $\{\frac{2}{\sqrt{3}}, \frac{3}{\sqrt{5}}, \frac{4}{\sqrt{7}}, \frac{5}{\sqrt{9}}\}$
D. $\{\frac{2}{\sqrt{2}}, \frac{4}{\sqrt{5}}, \frac{8}{\sqrt{10}}, \frac{16}{\sqrt{17}}\}$
E. $\{\frac{2}{\sqrt{3}}, \frac{4}{\sqrt{5}}, \frac{8}{\sqrt{7}}, \frac{16}{\sqrt{19}}\}$

2. (5 points) Find the **ratio** of the geometric sequence

$$-4, \frac{8}{3}, \frac{-16}{9}, \frac{32}{27}, \ldots$$

A. $r = \frac{2}{3}$
B. $r = -\frac{3}{2}$
C. $r = \frac{3}{2}$
D. $r = -\frac{2}{3}$
E. $r = 0$

3. (5 points) Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+20}$ converge or diverge?

A. Diverges because $\displaystyle\lim_{n\to\infty} \frac{\sqrt{n}}{n+20} \neq 0$.
B. Diverges by the limit comparison test to $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.
C. Diverges because it is geometric and $|r| > 1$.
D. Converges by the limit comparison test to $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.
E. Converges because $\displaystyle\lim_{n\to\infty} \frac{\sqrt{n}}{n+20} = 0$.

4. (5 points) Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{5^n} + \left(\frac{3}{5}\right)^n$

A. 4

B. $\frac{15}{4}$

C. 2

D. $\frac{7}{4}$

E. This series diverges.

5. (5 points) Which of the following series converge?

A. $\sum_{n=3}^{\infty} \frac{n+5}{\sqrt{n^2-6}}$

B. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{100}$

C. $\sum_{n=1}^{\infty} \frac{4^n}{1+3^n}$

D. $\sum_{n=1}^{\infty} \frac{10}{n^{2/3}}$

E. None of the given series converge.

6. (5 points) What would you compare $\sum_{n=1}^{\infty} \frac{n-3}{\sqrt{n^4+5n}}$ to for a conclusive limit comparison test?

A. $\sum_{n=1}^{\infty} \frac{1}{n^4}$

B. $\sum_{n=1}^{\infty} \frac{1}{n^3}$

C. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

D. $\sum_{n=1}^{\infty} \frac{1}{n}$

E. The limit comparison test can't be used to understand convergence for this series.

7. (5 points) Does the series $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ converge or diverge?

A. Converges by the ratio test because $\lim_{n\to\infty} \frac{2}{n+1} = 0$**.**

B. Diverges by the divergence test because $\lim_{n\to\infty} \frac{(-2)^n}{n!} = \infty$.

C. Converges because the series is telescoping.

D. Converges by the divergence test because $\lim_{n\to\infty} \frac{(-2)^n}{n!} = 0$.

E. Diverges by the ratio test because $\lim_{n\to\infty} \frac{2^{n+1}}{n+1} = \infty$.

8. (5 points) Find the smallest value of N so that S_N approximates $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3+5}$ to within an error of at most .001.

A. $N = 5$

B. $N = 9$

C. $N = 20$

D. $N = 21$

E. $N = 49$

9. (5 points) What is the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{(n+4)^n}$?

A. $\{2\}$

B. $[-3, 7)$

C. $[-3, 7]$

D. $[\frac{9}{5}, \frac{11}{5}]$

E. $(-\infty, \infty)$

10. (5 points) Which power series represents the function $x^5 \cos(3x)$ on the interval $(-\infty, \infty)$?

A. $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+5}}{(2n+1)!}$

B. $\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n} x^{2n+5}}{(2n)!}$

C. $\sum_{n=0}^{\infty} (-3)^n x^{n+5}$

D. $\sum_{n=0}^{\infty} \frac{(-3)^n x^{2n-5}}{(2n)!}$

E. $\sum_{n=0}^{\infty} \frac{3^n x^{n+5}}{n!}$

Free Response Questions

11. Decide if the series converges or diverges. Clearly state which test(s) are used.

(a) (5 points) $\displaystyle\sum_{n=9}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}-7}$

Solution: Converges by the alternating series test, since the series is alternating, and $\frac{1}{\sqrt[3]{n}-7}$ decreases to zero.

(b) (5 points) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}+7}$

Solution: Diverges by Limit Comparison to $\sum \frac{1}{n^{1/3}}$, a divergent p-series.

$$\lim_{n\to\infty} \frac{\frac{1}{\sqrt[3]{n}+7}}{\frac{1}{n^{1/3}}} = \lim_{n\to\infty} \frac{n^{1/3}}{n^{1/3}+7} = 1$$

and $0 < 1 < \infty$.

12. Are the series absolutely convergent, conditionally convergent or divergent? Justify your answers.

(a) (5 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n 2n}{5n+7}$

Solution: Diverges by the Divergence Test since

$$\lim_{n\to\infty} \frac{2n}{5n+7} = \frac{2}{5} \neq 0.$$

(b) (5 points) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{3^n + n}$

Solution: Converges absolutely by comparison to $\sum(-\frac{2}{3})^n$ a convergent geometric series.

13. (10 points) Find the interval of convergence for the series. Hint: Show clearly where you test the endpoints of your interval.

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n6^n}$$

Solution: Using the Ratio Test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(x-3)^{n+1}}{(n+1)6^{n+1}}\cdot\frac{n6^n}{(x-3)^n}\right| = \lim_{n\to\infty}\left|(x-3)\cdot\frac{n}{n+1}\cdot\frac{1}{6}\right| = \left|\frac{x-3}{6}\right|$$

Thus $-3 < x < 9$.

If $x = -3$ then $\sum \frac{(-6)^n}{n6^n} = \sum \frac{(-1)^n}{n}$ converges by the alternating series test.

If $x = 3$ then $\sum \frac{6^n}{n6^n} = \sum \frac{1}{n}$ diverges, because it is a p-series with $p = 1$.

Thus the interval of convergence is $[-3, 9)$.

14. (a) (5 points) Write a Taylor series centered at $x = 0$ for the function $f(x) = \dfrac{1}{1+5x^3}$.

Solution:

$$\frac{1}{1-(-5x^3)} = \sum_{n=0}^{\infty}(-5x^3)^n = \sum_{n=0}^{\infty}(-5)^n x^{3n}$$

(b) (5 points) Use your answer in (a) to help find the series for $g(x) = \dfrac{x^2}{(1+5x^3)^2}$ centered at $x = 0$. Hint: First compute $f'(x)$.

Solution:

$$f(x) = \sum_{n=0}^{\infty}(-5)^n x^{3n}$$

$$f'(x) = \frac{-15x^2}{(1+5x^3)^2} = \sum_{n=1}^{\infty}(-5)^n \cdot 3n \cdot x^{3n-1}$$

Thus,

$$g(x) = \frac{-1}{15}\sum_{n=1}^{\infty}(-5)^n \cdot 3n \cdot x^{3n-1}$$

15. (a) (4 points) Write the Maclaurin series, i.e., the Taylor Series centered at $x = 0$, for $f(x) = \cos(5x)$.

Solution:

$$f(x) = \sum_{n=0}^{\infty} \frac{(5x)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{5^{2n}x^{2n}}{(2n)!}$$

(b) (6 points) Write the first four terms of the Taylor series centered at $x = 3$ for $g(x)$, given that $g(3) = 10, g'(3) = 2, g''(3) = 7$, and $g'''(3) = -1$.

Solution:

$$g(x) = 10 + 2(x-3) + \frac{7}{2}(x-3)^2 + \frac{-1}{3!}(x-3)^3 + \ldots$$