**MA 114 — Calculus II** Spring 2015
**Exam 2** March 10, 2015

**Name:** _____

**Section:** _____

**Last 4 digits of student ID #:** _____

- No books or notes may be used.
- Turn off all your electronic devices and do not wear ear-plugs during the exam.
- You may use a calculator, but not one which has symbolic manipulation capabilities or a QWERTY keyboard.
- Additional blank sheets for scratch work are available upon request.
- **Multiple Choice Questions:** Record your answers on the right of this cover page by marking the box corresponding to the correct answer.
- **Free Response Questions:** Show all your work on the page of the problem. Clearly indicate your answer and the reasoning used to arrive at that answer.

**Multiple Choice Answers**

| Question | | | | | |
|---|---|---|---|---|---|
| 1 | A | ~~B~~ | C | D | E |
| 2 | A | B | C | ~~D~~ | E |
| 3 | A | B | ~~C~~ | ~~D~~ | E |
| 4 | A | B | ~~C~~ | D | E |
| 5 | ~~A~~ | B | C | D | E |
| 6 | A | B | C | D | ~~E~~ |
| 7 | A | B | C | D | ~~E~~ |

**Exam Scores**

| Question | Score | Total |
|---|---|---|
| MC | | 28 |
| 8 | | 14 |
| 9 | | 15 |
| 10 | | 15 |
| 11 | | 13 |
| 12 | | 15 |
| Total | | 100 |

**Unsupported answers for the free response questions may not receive credit!**

14 pts

**Free Response Questions: Show your work!**

8. Find the first four terms of the Taylor series for $f(x) = (1-x)^{-1/2}$ centered at 0. (Hint: you are looking for a polynomial of degree 3.)

$f'(x) = (-1/2)(1-x)^{-3/2}(-1) = \frac{1}{2}(1-x)^{-3/2}$

$f'(0) = \frac{1}{2}(1-0)^{-3/2} = 1/2$

$f''(x) = \frac{1}{2}(-3/2)(1-x)^{-5/2}(-1) = \frac{3}{4}(1-x)^{-5/2}$

$f''(0) = \frac{3}{4}(1-0)^{-5/2} = 3/4$

$f'''(x) = \frac{3}{4}\cdot(-5/2)(1-x)^{-7/2}(-1) = \frac{15}{8}(1-x)^{-7/2}$

$f'''(0) = \frac{15}{8}(1-0)^{-7/2} = 15/8$

} 9 pts - 3 pts each derivative

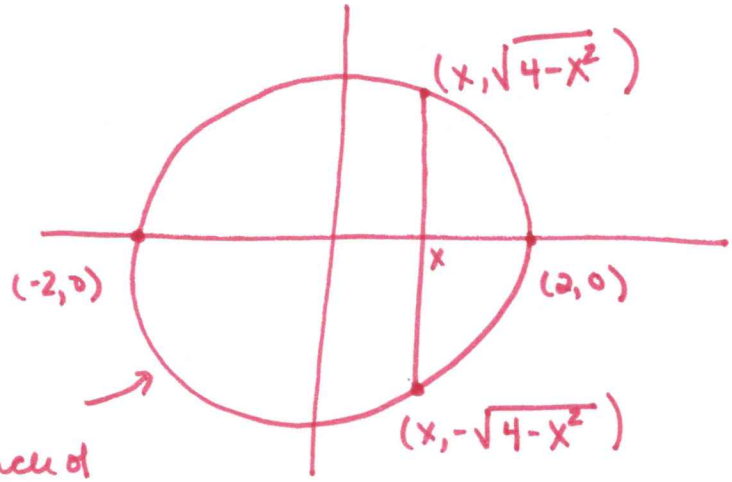
$T_3(x) = f(0) + \frac{f'(0)}{1!}(x-0) + \frac{f''(0)}{2!}(x-0)^2 + \frac{f'''(0)}{3!}(x-0)^3$ } 2 pts

$= 1 + \frac{1}{2}x + \frac{3}{4}\cdot\frac{1}{2}x^2 + \frac{15}{8}\cdot\frac{1}{6}x^3$ } 3 pts

15 pts

**Free Response Questions: Show your work!**

**9.** Find the volume of the solid whose base is the circle $x^2 + y^2 = 2^2$ and the cross sections perpendicular to the $x$-axis are squares.




the cross section area at slice $x$ is

$(\sqrt{4-x^2} - -\sqrt{4-x^2})^2$

$= (2\sqrt{4-x^2})^2 = 4(4-x^2)$ } 5 pts

endpts of integration $\pm 2$ } 2 pts

$$\text{volume} = \int_{-2}^{2} 4(4-x^2)\,dx \quad \} \text{ 4 pts}$$

$$= 4\left(4x - \tfrac{1}{3}x^3 \Big|_{-2}^{2}\right)$$

$$= 4\left(8 - \tfrac{1}{3}\cdot 2^3 - 4(-2) + \tfrac{1}{3}(-2)^3\right)$$

$$= 4\left(8 - \tfrac{8}{3} + 8 - 8/3\right) = 4\left(16 - 16/3\right)$$

$$= 4 \cdot \frac{32}{3}$$

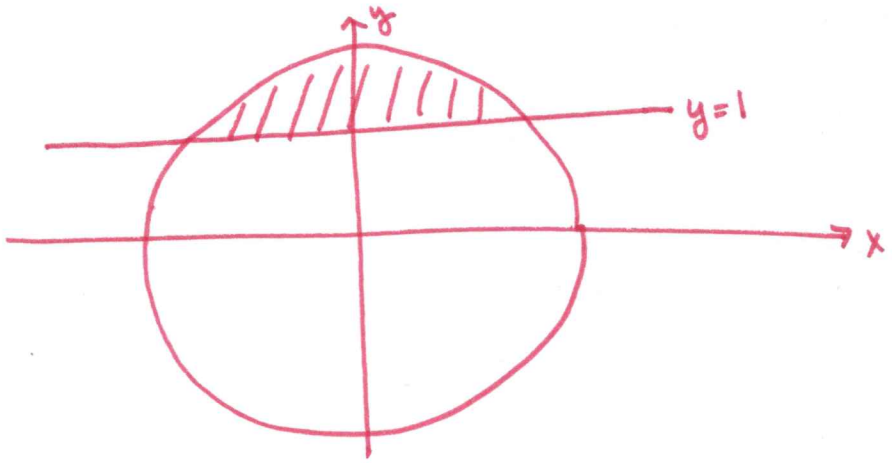
} 4 pts

15 pts

**Free Response Questions: Show your work!**

**10.** Consider the region between the circle $x^2 + y^2 = 4$ and the line $y = 1$ and above the $x$-axis.

(a) Graph this region.




3 pts

(b) Use the disk/washer method to find the volume of the solid given by revolving this region around the $x$-axis.

endpts $x^2+1^2=4$ $x^2=3$ $x=\pm\sqrt{3}$ } 2 pts

inside radius $1$
outside radius $\sqrt{4-x^2}$ } 2 pts (oh if this is not made explicit)

$$\text{Volume} = \int_{-\sqrt{3}}^{\sqrt{3}} \pi\left(\left(\sqrt{4-x^2}\right)^2 - 1^2\right) dx \quad \} 4 \text{ pts}$$

$$= \int_{-\sqrt{3}}^{\sqrt{3}} \pi\left(4 - x^2 - 1\right) dx = \int_{-\sqrt{3}}^{\sqrt{3}} \pi\left(3 - x^2\right) dx$$

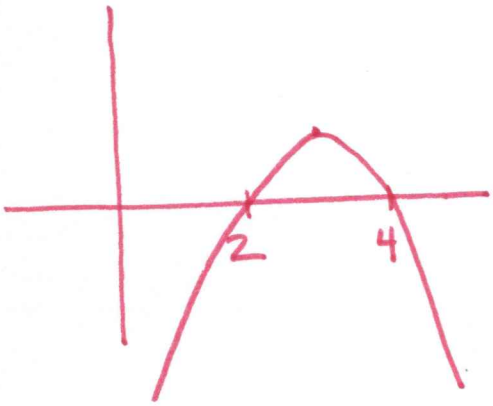
$$= \pi\left(3x - \frac{1}{3}x^3 \Big|_{-\sqrt{3}}^{\sqrt{3}}\right) = \pi\left(3\sqrt{3} - \frac{(\sqrt{3})^3}{3} - 3(-\sqrt{3}) + \frac{(-\sqrt{3})^3}{3}\right)$$

$$= \pi\left(6\sqrt{3} - \frac{2(\sqrt{3})^3}{3}\right) = \pi\left(6\sqrt{3} - 2\sqrt{3}\right) = 4\pi\sqrt{3} \quad \} 4 \text{ pts}$$

13 pts

**Free Response Questions: Show your work!**

**11.** Use the shell method to find the volume of the solid given by revolving the region between the graphs of $y = -x^2 + 6x - 8$ and $y = 0$ around the $y$-axis.




endpts $0 = -x^2 + 6x - 8$
$= (-1)(x-4)(x-2)$
endpts are 2, 4
} 2pts

cylinders have base at $y = 0$
top at $y = -x^2 + 6x - 8$
} 2pts

Volume $= \int_2^4 2\pi x(-x^2+6x-8)\,dx$ } 5pts

$= 2\pi \int_2^4 (-x^3+6x^2-8x)\,dx$

$= 2\pi \left(\frac{-1}{4}x^4 + 2x^3 - 4x^2 \Big|_2^4\right)$

$= 2\pi\left(-\frac{1}{4}\cdot 4^4 + 2\cdot 4^3 - 4\cdot 4^2 + \frac{1}{4}2^4 - 2\cdot 2^3 + 4\cdot 2^2\right)$

$= 2\pi\left(-4^3 + 2\cdot 4^3 - 4^3 + 4 - 2^4 + 2^4\right) = 8\pi$

} 4pts

15 pts

**Free Response Questions: Show your work!**

**12.** (a) Compute $\int \tan^3 x \sec^2 x dx$

$u = \tan x$
$du = \sec^2 x\, dx$ } 2pts

$\int \tan^3 x \sec^2 x\, dx = \int u^3 du = \frac{1}{4}u^4 + C$
$= \frac{1}{4}\tan^4 x + C$ } 3pts

(b) What is the volume of the solid given by revolving the region under $f(x) = \sin^{3/2} x$ and above $[0, \pi]$ around the $x$-axis?

endpts $0, \pi$ } 1pt

$\text{volume} = \int_0^\pi \pi (\sin^{3/2} x)^2 dx$ } 3pts

$= \pi \int_0^\pi \sin^3 x\, dx = \pi \int_0^\pi (1 - \cos^2 x) \sin x\, dx$ } 3pts

$= \pi \int_0^\pi (\sin x - \cos^2 x \sin x)\, dx$
$= \pi \left(-\cos x + \frac{1}{3}\cos^3 x \Big|_0^\pi\right)$
$= \pi\left(-(-1) + \frac{1}{3}(-1)^3 + (1) - \frac{1}{3}(1)^3\right)$
$= \pi(2 - 2/3) = 4\pi/3$ } 3pts