Problem 1.

5. **(5 points)** local/rmb-problems/e3/arc-length-num.pg

Find the length of the curve $y = \frac{2}{3}x^{3/2}$ between $x = 8$ and $x = 24$.
The length is _____

Exact answers are preferred. Your answer must be correctly rounded to three decimal places, or more accurate.

Problem 2.

3. **(5 points)** local/rmb-problems/e3/volume-shells-mc.pg

A solid is formed by rotating the region enclosed by the curves $y = x^3$, $y = 0$, $x = 1$, and $x = 2$ about the y-axis. Select the integral which computes the resulting volume.

- A. $2\pi \int_1^2 x^4\,dx$
- B. $2\pi \int_1^2 x\sqrt{1+9x^4}\,dx$
- C. $2\pi \int_0^1 x^4\,dx$
- D. $\pi \int_1^2 x^6\,dx$
- E. $\pi \int_0^1 x^6\,dx$

Problem 3.

6. **(5 points)** local/rmb-problems/e3/surface-area-2-mc.pg

The graph of $f(x) = x^2$ between the points $(2,4)$ and $(3,9)$ is rotated about the x-axis. Select the integral which computes the area of the resulting surface.

- A. $2\pi \int_2^3 x\sqrt{1+4x^2}\,dx$
- B. $2\pi \int_4^9 x^2\sqrt{1+x^4}\,dx$
- C. $2\pi \int_2^3 x^2\sqrt{1+4x^2}\,dx$
- D. $2\pi \int_4^9 x\sqrt{1+x^4}\,dx$
- E. $2\pi \int_2^3 x\sqrt{1+x^4}\,dx$

Problem 4.

8. **(5 points)** local/rmb-problems/e3/center-of-mass-num.pg

Three equal masses are placed at the points $(-4,-3)$, $(4,-3,)$, and $(0,3)$. Find the coordinates $(\bar{x},\bar{y})$ of the center of mass.
$\bar{x} =$ ____, $\bar{y} =$ ____.

Exact answers are preferred. Your answer should be correctly rounded to three decimal places, or more accurate.

Problem 5.

4. **(5 points)** local/rmb-problems/e3/washers-2-mc.pg

Let T be the triangle that is enclosed by the lines with equations $y = x$, $y = 2x-1$ and $x = 3$. We rotate the triangle T about the x-axis to obtain a solid of rotation S. Which of the following integrals computes the volume of the solid S?

- A. $\pi \int_1^3 ((2x-1)^2 - 3^2)\,dx$
- B. $\pi \int_1^3 (3^2 - x^2)\,dx$
- C. $\pi \int_1^3 ((2x-1)^2 - x^2)\,dx$
- D. $\pi \int_1^3 (x-1)^2\,dx$
- E. $\pi \int_1^5 ((2x-1)^2 - x^2)\,dx$

Problem 6.

2. **(5 points)** local/rmb-problems/e3/vol-slice-num.pg

A solid lies between $x = 2$ and $x = 5$. The cross-section at x is a circle with radius $r = 7x^2$. Find the volume of the solid.
The volume is _____
Exact answers are preferred. Your answer should be correctly rounded to three decimal places, or more accurate.

Problem 7.

7. **(5 points)** local/rmb-problems/e3/moment-mc.pg

Which of the following integrals represents the y-moment M_y of a thin plate that covers the region enclosed by the graphs $f(x) = x^2 - 4x + 6$ and $g(x) = x + 2$? The density of the plate is $\rho = 3$.

- A. $M_y = \int_1^4 (-x^2+5x-4)\,dx$
- B. $M_y = 3\int_1^4 x(-x^2+5x-4)\,dx$
- C. $M_y = 3\int_1^4 (-x^2+5x-4)\,dx$
- D. $M_y = \frac{3}{2}\int_1^4 ((2+x)^2-(x^2-4x+6)^2)\,dx$
- E. $M_y = 3\int_1^4 x(-x^2+3x-8)\,dx$

Problem 8.

1. **(5 points)** `local/rmb-problems/e3/average-num.pg`

Find the average value of the function $\sec^2(x)$ on the interval $[-\pi/6, \pi/4]$.
The average value is _____

Exact answers are preferred. Your answer should be correctly rounded to three decimal places, or more accurate.

This is the free response part of Exam 3. There are 3 questions, each worth 20 points. Please write your solutions in full, clearly indicating each step leading to the final answer. Omitting details will result in a lower grade.

Question 1. (a) Find the average value f_{ave} of the function $f(x) = \sin^2(x)$ on the interval $[0, \pi]$.

(b) Find **all** the values c in $[0, \pi]$ satisfying $f(c) = f_{\text{ave}}$.

(next page)

Question 2. Let $\mathcal{R}$ be the part of the disk $x^2 + y^2 \leq 4$ that lies above the line $y = 1$. Find the volume of the solid of revolution $\mathcal{S}$ obtained by rotating $\mathcal{R}$ about the x-axis. Clearly state which method (washer or cylindrtical shells) you are using.

(next page)

Question 3. Find the centroid of the region in the first quadrant of the xy-plane bounded by the curves $y = x^3$ and $x = y^3$.

(end of exam questions)