Exam 4

Name: ______________________ Section: ____________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. You are allowed to use notes on a single piece of 8.5" x 11" paper, front and back, including formulas and theorems. **You are required to turn this page in with your exam.** You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS). Absolutely no communication device use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1 (A) (B) (C) (D) (E) **6** (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E) **7** (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E) **8** (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E) **9** (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E) **10** (A) (B) (C) (D) (E)

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

Multiple Choice Questions

1. (5 points) Find $\int x^2 \ln(x)\ dx$.

   A. $x^2 \ln(x) - \frac{1}{3}x^3 + C$

   B. $\frac{1}{6}x^3 \ln(x) + C$

   C. $\frac{1}{3}x^3 \ln(x) - \frac{1}{9}x^3 + C$

   D. $x + 2x \ln(x) + C$

   E. $x^3 \ln(x) + \frac{1}{3}x^3 + C$

2. (5 points) Find the center of the ellipse with equation $y^2 + 4y + x^2 + 3x = 1$.

   A. $(-\frac{3}{2}, -2)$

   B. $(\frac{3}{2}, -1)$

   C. $(3, -1)$

   D. $(\frac{9}{4}, 2)$

   E. $(-\frac{9}{4}, -3)$

3. (5 points) Which of the following **sequences** converge?

   A. $b_n = \frac{3^n}{5^n}$

   B. $c_n = \frac{16 + (-1)^n n}{n^2}$

   C. $a_n = \sin\left(\frac{1}{n}\right)$

   D. None of the above.

   E. All of the above.

4. (5 points) Which of the following **series** converge?

   A. $\displaystyle\sum_{n=10}^{\infty} \frac{n+1}{\sqrt{n^2-1}}$

   B. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n+2)^{\frac{3}{2}}}$

   C. $\displaystyle\sum_{n=1}^{\infty} \frac{2n-1}{2n+1}$

   D. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n^2+3n)^{\frac{5}{2}}}$

   E. None of the above series converge.

5. (5 points) Consider the curve C parametrized by $x(t) = t^3 + 1$ and $y(t) = t^2 + t - 6$. Find the slope of the tangent line to C at $(2, -4)$.

   A. 6

   B. 1

   C. $\frac{2}{3}$

   D. $\frac{2}{9}$

   E. $\frac{4}{3}$

6. (5 points) Evaluate $\displaystyle\int_0^{\infty} \frac{1}{(x+2)^3}\,dx$

   A. $\frac{1}{8}$

   B. $\frac{1}{3}$

   C. 0

   D. $-\frac{1}{4}$

   E. This integral diverges

7. (5 points) Find the sum of the series $\sum_{n=1}^{\infty}\left[\left(\frac{2}{3}\right)^n - \left(\frac{1}{4}\right)^n\right]$

A. $\frac{1}{3}$

B. $\frac{5}{7}$

C. 0

D. This series is divergent.

E. $\frac{5}{3}$

8. (5 points) Find the center of mass of the system of particles given by a mass of 2 grams at $(-2, 0)$, a mass of 5 grams at $(7, 1)$, and a mass of 3 grams at $(1, 5)$.

A. $(4, 2)$

B. $(2, 4)$

C. $(\frac{34}{10}, 2)$

D. $(2, \frac{37}{12})$

E. $(0, \frac{27}{12})$

9. (5 points) Which of the following is the equation of a circle with center $(1, 2)$ and radius 2?

   A. $x^2 - 2x + y^2 - 4y + 1 = 0$

   B. $x^2 + 2x + y^2 - 4y + 1 = 0$

   C. $x^2 - 2x - y^2 - 4y + 1 = 0$

   D. $9x^2 - 2x + 4y^2 - 4y + 1 = 0$

   E. $2y + 4x^2 - 4x + 1 = 0$

10. (5 points) For any constant C, the function $y(x) = Ce^{\frac{1}{2}x^2} + 1$ is a solution to the differential equation $y' = x(y - 1)$. The unique solution satisfying $y(2) = 2$ is

    A. $y(x) = 2e^{\frac{1}{2}x^2}$

    B. $y(x) = e^{\frac{1}{2}x^2} + 1$

    C. $y(x) = e^{\frac{1}{2}x^2} + 2$

    D. $y(x) = e^{-2}e^{\frac{1}{2}x^2} + 1$

    E. $y(x) = 2e^{\frac{1}{2}x^2} - 2$

Free Response Questions

11. A parametric curve C is given by $x(t) = t^2 + 1$ and $y(t) = t^4 + t^2$ for $0 \leq t \leq 2$.

(a) (4 points) Set up an integral which computes the arc length of C.

(b) (6 points) Eliminate the t parameter to find a function $f(x)$ with the property that points on C satisfy $y = f(x)$.

12. (a) (5 points) Find the Taylor series of the function $\dfrac{x}{1-\frac{2}{3}x^2}$ centered at 0.

(b) (5 points) Find the radius of convergence for the series $\displaystyle\sum_{n=1}^{\infty} \frac{5^n(n+1)(x-3)^n}{n+7}$.

13. Consider the polar curve C defined by the equation $r = 1 + \cos(2\theta)$.

(a) (8 points) Find an equation for the tangent line to C at the point defined by the angle $\theta = \frac{\pi}{4}$.

(b) (2 points) Set up an integral which computes the area between C and the origin for $0 \leq \theta \leq \frac{\pi}{2}$.

14. (a) (6 points) Set up the integral for the volume of a solid obtained by revolving the region between the graph of $f(x) = 3x^2 - x^3$ and the x-axis **around the y-axis.** (Hint: use the shell method.)

(b) (4 points) Evaluate the integral in part (a) to find the volume of the solid of revolution.

15. (10 points) Using the method of partial fractions, compute

$$\int \frac{x}{(x-1)(x^2+1)}\,dx.$$