Exam 4

Name: ______________________________ Section: ______________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. You are allowed to use notes on a single piece of 8.5" x 11" paper, front and back, including formulas and theorems. **You are required to turn this page in with your exam.** You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS). Absolutely no communication device use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **6** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

2 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **7** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

3 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **8** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

4 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **9** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

5 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ **10** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

Multiple Choice Questions

1. (5 points) Evaluate $\int \sin^6(x)\cos^3(x)\,dx$.

A. $\frac{1}{7}\sin^7(x) - \frac{1}{9}\sin^9(x) + C$

B. $\frac{1}{7}\sin^7(x) \cdot \frac{1}{4}\cos^4(x) + C$

C. $\sin^6(x) - \sin^8(x) + C$

D. $\frac{1}{6}x - \frac{1}{2}\cos^4(2x) + \frac{1}{4}\cos^4(x) + C$

E. $\frac{1}{7}\sin^7(x)(1 - \frac{1}{3}\sin^3(x)) + C$

2. (5 points) What is the form of the partial fraction decomposition of

$$\frac{x+4}{x^3(x^2+11)}?$$

A. $\frac{A}{x^3} + \frac{Bx+C}{x^2+11}$

B. $\frac{A}{x^3} + \frac{B}{x^2+11}$

C. $\frac{A}{x} + \frac{B}{x^3} + \frac{Cx+D}{x^2+11}$

D. $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+11}$

E. $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^2+11} + \frac{E}{(x^2+11)^2}$

3. (5 points) Which of the following is equal to the integral $\int \frac{1}{x^2\sqrt{25-x^2}}dx$ after making the substitution $x = 5\sin\theta$?

   A. $\int \frac{1}{25}\csc^2\theta\, d\theta$

   B. $\int \frac{1}{625\sin^2\theta\cos\theta}\, d\theta$

   C. $\int -\frac{1}{25}\cot\theta\, d\theta$

   D. $\int \frac{\cos\theta}{5\sin^2\theta(5-5\sin\theta)}\, d\theta$

   E. $\int \frac{d\theta}{25\sin^2\theta(5-5\sin\theta)}$

4. (5 points) Use the **trapezoidal rule** with $n = 3$ intervals to approximate $\int_1^7 \sqrt{1+x^3}\, dx$.

   A. $\frac{1}{2}(\sqrt{1+(1)^3} + 2\sqrt{1+(3)^3} + 2\sqrt{1+(5)^3} + \sqrt{1+(7)^3})$

   B. $\sqrt{1+(1)^3} + 2\sqrt{1+(3)^3} + 2\sqrt{1+(5)^3} + \sqrt{1+(7)^3}$

   C. $\frac{2}{3}(\sqrt{1+(1)^3} + 4\sqrt{1+(3)^3} + 2\sqrt{1+(5)^3} + \sqrt{1+(7)^3})$

   D. $\frac{3}{2}(\sqrt{1+(1)^3} + 2\sqrt{1+(4)^3} + \sqrt{1+(7)^3})$

   E. $2\sqrt{1+(2)^3} + 2\sqrt{1+(4)^3} + 2\sqrt{1+(6)^3}$

5. (5 points) Find the sum of the series $\sum_{n=0}^{\infty} 4\left(\frac{-2}{3}\right)^n$.

   A. 12

   B. $\frac{3}{11}$

   C. $\frac{12}{5}$

   D. $\frac{3}{2}$

   E. This series diverges.

6. (5 points) What would you compare $\sum_{n=2}^{\infty} \frac{n+6}{\sqrt{n^5+7n}}$ to for a conclusive limit comparison test?

   A. $\sum_{n=2}^{\infty} \frac{1}{n^{3/2}}$

   B. $\sum_{n=2}^{\infty} \frac{1}{n^{5/2}}$

   C. $\sum_{n=2}^{\infty} \frac{1}{n^3}$

   D. $\sum_{n=2}^{\infty} \frac{1}{n^4}$

   E. $\sum_{n=2}^{\infty} n^{4/5}$

7. (5 points) A surface is created by rotating the graph of $f(x) = 3 + x^3$ from $x = 1$ to $x = 4$ around the x-axis. Which integral computes the **surface area** of the resulting surface?

   A. $\int_1^4 2\pi x\sqrt{1+3x^2}dx$

   B. $\int_1^4 2\pi(3+x^3)\sqrt{1+3x^2}dx$

   C. $\int_1^4 2\pi x\sqrt{1+(3+x^3)^2}dx$

   D. $\int_1^4 2\pi(3+x^3)\sqrt{1+(3+x^3)^2}dx$

   E. $\int_1^4 2\pi(3+x^3)\sqrt{1+9x^4}dx$

8. (5 points) Consider the curve C parametrized by $x(t) = 2t - 7$ and $y(t) = 3t^2 - t + 2$. Find the slope of the tangent line to C at $(-1, 26)$.

A. 3

B. 8

C. -26

D. $\frac{17}{2}$

E. $\frac{155}{2}$

9. (5 points) Which of the following integrals computes the **arc length** of the parametric curve $x(t) = 3t + 1$, $y(t) = 4 - t^2$, $-2 \le t \le 0$?

A. $\int_{-2}^{0} t\sqrt{9 - 2t}dt$

B. $\int_{-2}^{0} \sqrt{9 + 4t^2}dt$

C. $\int_{-2}^{0} \sqrt{1 + \left(\frac{2t}{3}\right)^2}dt$

D. $\int_{-2}^{0} 2\pi(4 - t^2)\sqrt{1 + (2t)^2}dt$

E. $\int_{-2}^{0} \sqrt{1 + 4t^2}dt$

10. (5 points) Find the equation of the ellipse with foci $(0, \pm 10)$ and two vertices $(\pm 5, 0)$.

A. $\frac{x^2}{25} + \frac{y^2}{100} = 1$

B. $\frac{x^2}{125} + \frac{y^2}{25} = 1$

C. $\frac{x^2}{25} + \frac{y^2}{125} = 1$

D. $\frac{x^2}{25} + \frac{y^2}{75} = 1$

E. $\frac{(x-5)^2}{25} + \frac{(y-10)^2}{100} = 1$

Free Response Questions

11. (a) (5 points) Compute $\int x\cos(3x)\ dx$.

Solution: Use integration by parts with $u = x$, $du = dx$, $V = \frac{1}{3}\sin(3x)$, and $dV = \cos(3x)dx$. Then

$$\int x\cos(3x)\ dx = \frac{1}{3}x\sin(3x) - \int \frac{1}{3}\sin(3x)\ dx = \frac{1}{3}x\sin(3x) + \frac{1}{9}\cos(3x) + C$$

(b) (5 points) Find the Taylor series for the function $\dfrac{x^2}{1-x^6}$ centered at 0.

Solution:

$$x^2\frac{1}{1-x^6} = x^2\sum_{n=0}^{\infty}(x^6)^n = x^2\sum_{n=0}^{\infty}x^{6n} = \sum_{n=0}^{\infty}x^{6n+2}.$$

12. (10 points) Find the **interval** of convergence for the power series:

$$\sum_{n=1}^{\infty} \frac{2^n (x-1)^n}{n}.$$

Be sure to show all work necessary to justify your answer.

Solution: We use the ratio test to establish the center and radius of convergence, then we test the endpoints. The ratio test goes like this:

$$\lim_{n\to\infty} \mid \frac{a_{n+1}}{a_n} \mid = \lim_{n\to\infty} \mid \frac{2^{n+1}(x-1)^{n+1}}{n+1} \frac{n}{2^n (x-1)^n} \mid = \lim_{n\to\infty} \mid \frac{2^{n+1}}{2^n} \frac{n}{n+1} \frac{(x-1)^{n+1}}{(x-1)^n} \mid .$$

Taking the limit we obtain $2 \mid x-1 \mid$.

For this to be < 1 we must have $|x-1| < \frac{1}{2}$ or $-\frac{1}{2} < x - 1 < \frac{1}{2}$ or $\frac{1}{2} < x < \frac{3}{2}$.

Now we test the end points.

At $x = \frac{1}{2}$ we get the series

$$\sum_{n=1}^{\infty} \frac{2^n (\frac{1}{2} - 1)^n}{n} = \sum_{n=1}^{\infty} \frac{2^n (-\frac{1}{2})^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n},$$

which converges by the alternating series test.

At $x = \frac{3}{2}$ we get the series

$$\sum_{n=1}^{\infty} \frac{2^n (\frac{3}{2} - 1)^n}{n} = \sum_{n=1}^{\infty} \frac{2^n (\frac{1}{2})^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n},$$

which diverges, because it is a p-series with $p = 1$.

We conclude that the interval of convergence is $[\frac{1}{2}, \frac{3}{2})$.

13. Let S be the solid obtained by rotating the region bounded by the curves $y = x^2$ and $y = 5x$ about the x-axis.

(a) (5 points) Set up the integral that computes the volume of S using the **disk/washer** method.

Solution:

$$\int_0^5 \pi((5x)^2 - (x^2)^2)dx$$

(b) (5 points) Set up the integral that computes the volume of S using the **cylindrical shells** method.

Solution:

$$\int_0^{25} 2\pi y(\sqrt{y} - \frac{1}{5}y)dy$$

14. Consider the polar curve C defined by $r = 2 + \cos\theta$.

(a) (5 points) Set up (but do not evaluate) an integral which computes the area between C and the origin for $0 \leq \theta \leq \pi$.

Solution: $A = \int \frac{1}{2} r^2 d\theta = \int_0^\pi \frac{1}{2}(2+\cos\theta)^2 \; d\theta$

(b) (5 points) Set up (but do not evaluate) an integral which computes the **arc length** of C for $0 \leq \theta \leq \pi$.

Solution:

$$L = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta = \int_0^\pi \sqrt{(2+\cos\theta)^2 + (-\sin\theta)^2} d\theta$$

15. (a) (5 points) Write the equation of the parabola which has vertex $(4, 0)$ and focus $(-2, 0)$.

Solution: This parabola open to the left, so the general equation is

$$x = -\frac{1}{4p}(y-k)^2 + h.$$

We see that $p = 6$, the distance from the vertex to the focus. Moreover, $(h, k) = (4, 0)$, so the equation becomes

$$x = -\frac{1}{24}y^2 + 4.$$

(b) (5 points) Find the **center** and **vertices** of the hyperbola defined by

$$\frac{(y-1)^2}{4} - \frac{(x+5)^2}{16} = 1.$$

Solution: The hyperbola opens up/down with center $(-5, 1)$. Since $b = 2$, then the vertices are $(-5, 3)$ and $(-5, -1)$.