MA 114 — Calculus II Spring 2013
Final Exam 1 May 2013

Name: ______________________________

Section: __________

Last 4 digits of student ID #: __________

This exam has six multiple choice questions (six points each) and five free response questions with points as shown. Additional blank sheets are available if necessary for scratch work. No books or notes may be used. Turn off your cell phones and do not wear ear-plugs during the exam. You may use a calculator, but not one which has symbolic manipulation capabilities.

On the multiple choice problems:

1. You must give your *final answers* in the *multiple choice answer box* on the front page of your exam.
2. Carefully check your answers. No credit will be given for answers other than those indicated on the *multiple choice answer box*.

On the free response problems:

1. Clearly indicate your answer and the reasoning used to arrive at that answer *(unsupported answers will not receive credit)*.
2. Give exact answers, rather than decimal approximations to the answer (unless otherwise stated).

Each free response question is followed by space to write your answer. Please write your solutions neatly in the space below the question. You are not expected to write your solution next to the statement of the question.

Multiple Choice Answers

Question					
1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E

Exam Scores

Question	Score	Total
MC		36
7		14
8		12
9		12
10		12
11		14
Total		100

Record the correct answer to the following problem on the front page of this exam.

---

(1) The polar coordinates (r, θ) of the point P with rectangular coordinates $(x, y) = (-\sqrt{3}, -1)$ can be expressed as:

A) $(2, \pi/6)$

B) $(-2, -\pi/6)$

C) $(2, -\pi/6)$

D) $(-2, 5\pi/6)$

E) $(-2, \pi/6)$

(2) The motion of a particle in the x-y plane is given parametrically by the equations

$$x(t) = 4t - \cos 4t, \qquad y(t) = 4t - \sin 4t, \qquad \text{for} \quad 0 \leq t \leq 2\pi,$$

where units of distance and time are in meters and seconds, respectively.

The speed (in m/s) of the particle at $t = \pi/4$ is:

A) 0

B) 1

C) 3/2

D) 2

E) 5/2

Record the correct answer to the following problem on the front page of this exam.

(3) Consider the ordinary differential equation $y'(t) = 2y(t)(3 - y(t))$. Which initial condition guarantees that $y(t) \to 3$ as $t \to +\infty$?

A) $y(0) = 0$

B) $y(0) = -2$

C) $y(0) = -0.5$

D) $y(0) = 1$

E) All of the above initial conditions

(4) Find the most general form of the anti-derivative of $x \sin(3x)$ by integration-by-parts. Below, C denotes an arbitrary constant.

A) $-x \sin(3x) + C$

B) $-(1/3) \sin(3x) - (1/9) \cos(3x) + C$

C) $-(x/3) \cos(3x) + (1/9) \sin(3x) + C$

D) $-x \cos(3x) + C$

E) None of the above.

Record the correct answer to the following problem on the front page of this exam.

(5) The coefficient of the x^2 term of the Maclaurin series (the Taylor series about $c = 0$) for $f(x) = \dfrac{e^{2x}}{1+x}$ is:

A) 1

B) 3

C) 5

D) 2

E) 4

(6) The volume of the solid of revolution obtained by rotating the graph of $f(x) = \sqrt{1 + 3x^2}$, for $x \in [0, 2]$, about the x-axis is:

A) 14π

B) 10π

C) $14\pi/3$

D) $13\pi/5$

E) None of the above

Free Response Questions: Show your work!

(7) (a) Find the Maclaurin series (the Taylor series about $c = 0$) of the function $f(x) = \dfrac{1}{1+2x}$ and its radius of convergence.

(b) Find the Maclaurin series (the Taylor series about $c = 0$) of $\ln(1+2x)$ and its radius of convergence. **Hint**: Use the fact that $\displaystyle\int \frac{dx}{1+2x} = \frac{1}{2}\ln(1+2x) + C$.

Free Response Questions: Show your work!

(8) Compute the following indefinite integral. Clearly show all of your work.

$$\int \frac{1}{x^3\sqrt{x^2-4}}\,dx$$

Free Response Questions: Show your work!

(9) Find the arc length of the parameterized curve $(x(t), y(t)) = (e^t \cos t, e^t \sin t)$ for $t \in [0, \ln 5]$.

(10) (a) Show that the following series converges absolutely or diverges. Clearly state your argument.

$$\sum_{n=1}^{\infty} \frac{\sin^4 n}{n^{3/2}}$$

(b) Find the radius of convergence of the following power series.

$$\sum_{n=0}^{\infty} \frac{3^n}{(n+2)^3}(x-3)^n$$

(11) (a) Convert $x = 3$ to an equation in polar coordinates, and determine the polar coordinates of the points where the line $x = 3$ meets the circle $r = 4\cos\theta$.

(b) Express as an integral the area of the region with $x \geq 3$ and enclosed by the given line and circle, and evaluate the integral. (No credit will be given unless you find the required area by following the instructions.)

Trigonometric Identities

$$\sin^2(x) = \frac{1 - \cos(2x)}{2}$$

$$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\sin(x + y) = \sin(x)\cos(y) + \sin(y)\cos(x)$$

$$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\sin^2(x) + \cos^2(x) = 1$$

$$\tan^2(x) + 1 = \sec^2(x)$$