Exam 4

Name: _____   Section: _____

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. You are allowed to use notes on a single piece of 8.5" x 11" paper, front and back, including formulas and theorems. **You are required to turn this page in with your exam.** You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS). Absolutely no communication device use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show <u>all work</u> to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ   **6** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

2 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ   **7** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

3 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ   **8** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

4 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ   **9** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

5 Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ   **10** Ⓐ Ⓑ Ⓒ Ⓓ Ⓔ

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

Multiple Choice Questions

1. (5 points) Use Simpson's Rule with $n = 4$ intervals to approximate $\int_0^2 \sqrt{1 + x^2}\, dx$.

    A. $\frac{1}{4}(1\sqrt{1 + (0)^2} + 2\sqrt{1 + (\frac{1}{2})^2} + 2\sqrt{1 + (1)^2} + 2\sqrt{1 + (\frac{3}{2})^2} + 1\sqrt{1 + (2)^2})$

    B. $\frac{1}{6}(1\sqrt{1 + (0)^2} + 4\sqrt{1 + (\frac{1}{2})^2} + 2\sqrt{1 + (1)^2} + 4\sqrt{1 + (\frac{3}{2})^2} + 1\sqrt{1 + (2)^2})$

    C. $\frac{1}{6}(1\sqrt{1 + (0)^2} + 1\sqrt{1 + (\frac{1}{2})^2} + 1\sqrt{1 + (1)^2} + 1\sqrt{1 + (\frac{3}{2})^2} + 1\sqrt{1 + (2)^2})$

    D. $\frac{1}{2}(1\sqrt{1 + (0)^2} + 1\sqrt{1 + (\frac{1}{2})^2} + 1\sqrt{1 + (1)^2} + 1\sqrt{1 + (\frac{3}{2})^2} + 1\sqrt{1 + (2)^2})$

    E. $\frac{1}{2}(1\sqrt{1 + (0)^2} + 1\sqrt{1 + (\frac{1}{2})^2} + 1\sqrt{1 + (1)^2} + 1\sqrt{1 + (\frac{3}{2})^2})$

2. (5 points) Find the **focus** and the **directrix** of the parabola with equation $6x = y^2$.

    A. **Focus** $(0, 3)$ **Directrix** $y = -3$

    B. **Focus** $(0, 0)$ **Directrix** $y = -3$

    C. **Focus** $(0, \frac{3}{2})$ **Directrix** $x = \frac{3}{2}$

    D. **Focus** $(0, 0)$ **Directrix** $y = -12$

    E. **Focus** $(\frac{3}{2}, 0)$ **Directrix** $x = -\frac{3}{2}$

3. (5 points) Find the sum of the series $\displaystyle\sum_{n=2}^{\infty} \frac{3 + 2^n}{4^n}$.

     A. $\frac{3}{4}$

     B. $\frac{7}{12}$

     C. $\frac{9}{16}$

     D. $\frac{3}{2}$

     E. $\frac{11}{12}$

4. (5 points) Compute the indefinite integral $\displaystyle\int \sqrt{1 - x^2}\,dx$.

     A. $\frac{1}{2}(\ln(x + \sqrt{1 - x^2}) + x\sqrt{1 - x^2}) + C$

     B. $\frac{2}{3}(1 - x^2)^{\frac{3}{2}} + C$

     C. $(1 - x^2)^{\frac{3}{2}} + 2x\sqrt{1 - x^2} + C$

     D. $\frac{1}{2}(\arcsin(x) + x\sqrt{1 - x^2}) + C$

     E. $x - \frac{1}{3}x^3 + C$

5. (5 points) Consider the curve C parametrized by $x(t) = t - 3$ and $y(t) = t^2 + t + 1$. Find the slope of the tangent line to C at $(-1, 7)$.

    A. -2

    B. 0

    C. 5

    D. The slope is undefined at this point.

    E. 2

6. (5 points) What would you compare $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^4 + 1}}{n^5 + 2n}$ to for a conclusive limit comparison test?

    A. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$

    B. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$

    C. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^5}$

    D. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n}$

    E. The limit comparison test can't be used to understand convergence for this series.

7. (5 points) What is the form of the partial fraction decomposition of
$$\frac{x^2 - 2}{(x + 1)^2 (x^2 - x + 7)}?$$

A. $\dfrac{A}{x + 1} + \dfrac{B}{(x + 1)^2} + \dfrac{Cx + D}{x^2 - x + 7} + \dfrac{Ex + F}{(x^2 - x + 7)^2}$

B. $\dfrac{A}{x + 1} + \dfrac{B}{(x + 1)^2} + \dfrac{Cx + D}{x^2 - x + 7}$

C. $\dfrac{A}{x + 1} + \dfrac{Bx + C}{x^2 - x + 7}$

D. $\dfrac{A}{x + 1} + \dfrac{B}{(x + 1)^2} + \dfrac{C}{x^2 - x + 7}$

E. $\dfrac{A}{(x + 1)^2} + \dfrac{Bx + C}{x^2 - x + 7}$

8. (5 points) Find the coefficient B in the partial fraction decomposition
$$\frac{1}{(x)(x^2 + x + 1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + x + 1}$$

A. $B = 3$

B. $B = 0$

C. $B = -2$

D. $B = -1$

E. $B = \frac{1}{2}$

9. (5 points) Which of the following integrals computes the **arc length** of the parametric curve $x(t) = t^2$, $y(t) = t^3$, $1 \leq t \leq 2$?

     A. $\displaystyle\int_1^2 \sqrt{4 + 9t^2}\,dt$

     B. $\displaystyle\int_1^2 \sqrt{1 + t^2}\,dt$

     C. $\displaystyle\int_1^2 t\sqrt{4 + 9t^2}\,dt$

     D. $\displaystyle\int_1^2 (t - 1)\sqrt{t^2 - 1}\,dt$

     E. $\displaystyle\int_1^2 2\pi t\sqrt{1 + 9t^2}\,dt$

10. (5 points) A surface is created by rotating the graph of $f(x) = e^x - 1$ from $x = 0$ to $x = 15$ around the x-axis. What is the integral that computes the area of this surface?

     A. $\displaystyle\int_0^{15} 2\pi x(1 + e^{2x})\,dx$

     B. $\displaystyle\int_0^{15} 2\pi(e^x)\sqrt{1 + e^x}\,dx$

     C. $\displaystyle\int_0^{15} e^x - \sqrt{1 + e^x}\,dx$

     D. $\displaystyle\int_0^{15} 2\pi x\sqrt{e^x + e^{2x}}\,dx$

     E. $\displaystyle\int_0^{15} 2\pi(e^x - 1)\sqrt{1 + e^{2x}}\,dx$

Free Response Questions

11. (a) (5 points) Compute $\displaystyle\int x \ln(x) \, dx$.

(b) (5 points) Find the Taylor series for the function xe^x centered at 0.

12. (10 points) Find the **interval** of convergence for the power series:

$$\sum_{n=1}^{\infty} \frac{(x+1)^n}{5^n}.$$

Be sure to show all work necessary to justify your answer.

13. (a) (5 points) Find the foci of the ellipse defined by the equation

$$\frac{(y-1)^2}{4} + \frac{(x+1)^2}{9} = 1.$$

(b) (5 points) Find the foci of the hyperbola defined by the equation

$$\frac{y^2}{4} - \frac{x^2}{9} = 1.$$

14.   (a) (4 points) Set up an integral to compute the **arc length** of the polar curve $r = 2\cos(\theta)$ for $0 \leq \theta \leq \pi$.

     (b) (2 points) Compute the **arc length** of the polar curve $r = 2\cos(\theta)$ for $0 \leq \theta \leq \pi$.

     (c) (4 points) Find a Cartesian equation for the polar curve $r = 2\cos(\theta)$.

15. Let S be the solid obtained by rotating the region bounded by the circle $x^2 + y^2 = 1$ around the line $x = 2$.

    (a) (5 points) Set up the integral that computes the volume of S using the disk/washer method.

    (b) (5 points) Set up the integral that computes the volume of S using the cylindrical shells method.