Exam 4

Name: ______________________________ Section: ______________

Do not remove this answer page — you will return the whole exam. You will be allowed two hours to complete this test. You are allowed to use notes on a single piece of 8.5" x 11" paper, front and back, including formulas and theorems. **You are required to turn this page in with your exam.** You may use a graphing calculator during the exam, but NO calculator with a Computer Algebra System (CAS). Absolutely no communication device use during the exam is allowed.

The exam consists of 10 multiple choice questions and 5 free response questions. Record your answers to the multiple choice questions on this page by filling in the circle corresponding to the correct answer.

Show all work to receive full credit on the free response problems. It will also help you check your answers to show work on multiple choice problems.

Multiple Choice Questions

1 (A) (B) (C) (D) (E)

2 (A) (B) (C) (D) (E)

3 (A) (B) (C) (D) (E)

4 (A) (B) (C) (D) (E)

5 (A) (B) (C) (D) (E)

6 (A) (B) (C) (D) (E)

7 (A) (B) (C) (D) (E)

8 (A) (B) (C) (D) (E)

9 (A) (B) (C) (D) (E)

10 (A) (B) (C) (D) (E)

Multiple Choice	11	12	13	14	15	Total Score
50	10	10	10	10	10	100

Multiple Choice Questions

1. (5 points) Use Simpson's Rule with $n = 4$ intervals to approximate $\int_0^2 \sqrt{1+x^2}\, dx$.

   A. $\frac{1}{4}(1\sqrt{1+(0)^2} + 2\sqrt{1+(\frac{1}{2})^2} + 2\sqrt{1+(1)^2} + 2\sqrt{1+(\frac{3}{2})^2} + 1\sqrt{1+(2)^2})$

   B. $\frac{1}{6}(1\sqrt{1+(0)^2} + 4\sqrt{1+(\frac{1}{2})^2} + 2\sqrt{1+(1)^2} + 4\sqrt{1+(\frac{3}{2})^2} + 1\sqrt{1+(2)^2})$

   C. $\frac{1}{6}(1\sqrt{1+(0)^2} + 1\sqrt{1+(\frac{1}{2})^2} + 1\sqrt{1+(1)^2} + 1\sqrt{1+(\frac{3}{2})^2} + 1\sqrt{1+(2)^2})$

   D. $\frac{1}{2}(1\sqrt{1+(0)^2} + 1\sqrt{1+(\frac{1}{2})^2} + 1\sqrt{1+(1)^2} + 1\sqrt{1+(\frac{3}{2})^2} + 1\sqrt{1+(2)^2})$

   E. $\frac{1}{2}(1\sqrt{1+(0)^2} + 1\sqrt{1+(\frac{1}{2})^2} + 1\sqrt{1+(1)^2} + 1\sqrt{1+(\frac{3}{2})^2})$

2. (5 points) Find the **focus** and the **directrix** of the parabola with equation $6x = y^2$.

   A. **Focus** $(0, 3)$ **Directrix** $y = -3$

   B. **Focus** $(0, 0)$ **Directrix** $y = -3$

   C. **Focus** $(0, \frac{3}{2})$ **Directrix** $x = \frac{3}{2}$

   D. **Focus** $(0, 0)$ **Directrix** $y = -12$

   E. Focus $(\frac{3}{2}, 0)$ **Directrix** $x = -\frac{3}{2}$

3. (5 points) Find the sum of the series $\displaystyle\sum_{n=2}^{\infty} \frac{3+2^n}{4^n}$.

   A. $\frac{3}{4}$

   B. $\frac{7}{12}$

   C. $\frac{9}{16}$

   D. $\frac{3}{2}$

   E. $\frac{11}{12}$

4. (5 points) Compute the indefinite integral $\displaystyle\int \sqrt{1-x^2}dx$.

   A. $\frac{1}{2}(\ln(x+\sqrt{1-x^2}) + x\sqrt{1-x^2}) + C$

   B. $\frac{2}{3}(1-x^2)^{\frac{3}{2}} + C$

   C. $(1-x^2)^{\frac{3}{2}} + 2x\sqrt{1-x^2} + C$

   D. $\frac{1}{2}(\arcsin(x) + x\sqrt{1-x^2}) + C$

   E. $x - \frac{1}{3}x^3 + C$

5. (5 points) Consider the curve C parametrized by $x(t) = t - 3$ and $y(t) = t^2 + t + 1$. Find the slope of the tangent line to C at $(-1, 7)$.

   A. -2

   B. 0

   C. 5

   D. The slope is undefined at this point.

   E. 2

6. (5 points) What would you compare $\sum_{n=1}^{\infty} \frac{\sqrt{n^4+1}}{n^5+2n}$ to for a conclusive limit comparison test?

   A. $\sum_{n=1}^{\infty} \frac{1}{n^3}$

   B. $\sum_{n=1}^{\infty} \frac{1}{n^4}$

   C. $\sum_{n=1}^{\infty} \frac{1}{n^5}$

   D. $\sum_{n=1}^{\infty} \frac{1}{2n}$

   E. The limit comparison test can't be used to understand convergence for this series.

7. (5 points) What is the form of the partial fraction decomposition of

$$\frac{x^2-2}{(x+1)^2(x^2-x+7)}?$$

A. $\dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}+\dfrac{Cx+D}{x^2-x+7}+\dfrac{Ex+F}{(x^2-x+7)^2}$

B. $\dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}+\dfrac{Cx+D}{x^2-x+7}$

C. $\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2-x+7}$

D. $\dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}+\dfrac{C}{x^2-x+7}$

E. $\dfrac{A}{(x+1)^2}+\dfrac{Bx+C}{x^2-x+7}$

8. (5 points) Find the coefficient B in the partial fraction decomposition

$$\frac{1}{(x)(x^2+x+1)}=\frac{A}{x}+\frac{Bx+C}{x^2+x+1}$$

A. $B=3$

B. $B=0$

C. $B=-2$

D. $B=-1$

E. $B=\frac{1}{2}$

9. (5 points) Which of the following integrals computes the **arc length** of the parametric curve $x(t) = t^2$, $y(t) = t^3$, $1 \le t \le 2$?

   A. $\int_1^2 \sqrt{4+9t^2}dt$

   B. $\int_1^2 \sqrt{1+t^2}dt$

   C. $\int_1^2 t\sqrt{4+9t^2}dt$

   D. $\int_1^2 (t-1)\sqrt{t^2-1}dt$

   E. $\int_1^2 2\pi t\sqrt{1+9t^2}dt$

10. (5 points) A surface is created by rotating the graph of $f(x) = e^x - 1$ from $x = 0$ to $x = 15$ around the x-axis. What is the integral that computes the area of this surface?

    A. $\int_0^{15} 2\pi x(1+e^{2x})dx$

    B. $\int_0^{15} 2\pi(e^x)\sqrt{1+e^x}dx$

    C. $\int_0^{15} e^x - \sqrt{1+e^x}dx$

    D. $\int_0^{15} 2\pi x\sqrt{e^x+e^{2x}}dx$

    E. $\int_0^{15} 2\pi(e^x-1)\sqrt{1+e^{2x}}dx$

Free Response Questions

11. (a) (5 points) Compute $\int x \ln(x)\ dx$.

Solution: We use integration by parts; set $u = \ln(x), dv = x$ to get $du = \frac{1}{x}$, $v = \frac{1}{2}x^2$ and

$$\int x \ln(x) dx = \frac{1}{2}x^2 \ln(x) - \frac{1}{2}\int x dx =$$

$$\frac{1}{2}x^2 \ln(x) - \frac{1}{4}x^2 + C$$

(b) (5 points) Find the Taylor series for the function xe^x centered at 0.

Solution: The Taylor series for e^x centered at 0 is $\sum_{n=0}^{\infty} \frac{x^n}{n!}$. Multiplying through by x gives

$$xe^x = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}.$$

12. (10 points) Find the **interval** of convergence for the power series:

$$\sum_{n=1}^{\infty} \frac{(x+1)^n}{5^n}.$$

Be sure to show all work necessary to justify your answer.

Solution: We use the ratio test to establish the center and radius of convergence, then we test the endpoints. The ratio test goes like this:

$$\lim_{n\to\infty} \left| \frac{(x+1)^{n+1}}{5^{n+1}} \frac{5^n}{(x+1)^n} \right| = \frac{1}{5} \lim_{n\to\infty} |x+1| = \frac{1}{5}|x+1|.$$

For this to be < 1 we must have $|x+1| < 5$. We conclude that the series is centered at -1 with radius of convergence 5. Now we test the end points $-1+5 = 4$, $-1-5 = -6$. At -6 we get the series

$$\sum_{n=0}^{\infty} \frac{(-5)^n}{5^n} = \sum_{n=0}^{\infty} (-1)^n,$$

and at 4 we get the series

$$\sum_{n=0}^{\infty} \frac{5^n}{5^n} = \sum_{n=0}^{\infty} 1.$$

Both of these series diverge, we get $(-6, 4)$ as our interval of convergence.

13. (a) (5 points) Find the foci of the ellipse defined by the equation

$$\frac{(y-1)^2}{4} + \frac{(x+1)^2}{9} = 1.$$

Solution: The centroid of this ellipse is $(-1, 1)$, with vertices $(0, \pm 2)$ and $(\pm 3, 0)$. The distance from the foci to the centroid then satisfies $c^2 = 3^2 - 2^2 = 5$, so $c = \sqrt{5}$. The major axis of this ellipse is the line $y = 1$, parallel to the x-axis, so the foci are the points $(-1 + \sqrt{5}, 1)$, $(-1 - \sqrt{5}, 1)$.

(b) (5 points) Find the foci of the hyperbola defined by the equation

$$\frac{y^2}{4} - \frac{x^2}{9} = 1.$$

Solution: The vertices of this hyperbola are the points $(0, 2)$ and $(0, -2)$, in particular the major axis is the y-axis and the center is $(0, 0)$. The foci are at distance c from $(0, 0)$ along the y-axis, where $c^2 = 4 + 9 = 13$. As a consequence, the foci are $(0, \sqrt{13}), (0, -\sqrt{13})$.

14. (a) (4 points) Set up an integral to compute the **arc length** of the polar curve $r = 2\cos(\theta)$ for $0 \leq \theta \leq \pi$.

Solution: The arclength formula for $r = f(\theta)$ is $\int \sqrt{f^2 + (f')^2} d\theta$, we have:

$$(2\cos(\theta))' = -2\sin(\theta),$$

so we get:

$$\int_0^\pi \sqrt{(2\cos(\theta)^2 + (-2\sin(\theta)^2} d\theta$$

(b) (2 points) Compute the **arc length** of the polar curve $r = 2\cos(\theta)$ for $0 \leq \theta \leq \pi$.

Solution: The integral above amounts to:

$$2\int_0^\pi \sqrt{\cos^2(\theta) + \sin^2(\theta)} d\theta = 2\int_0^\pi d\theta = 2\pi.$$

(c) (4 points) Find a Cartesian equation for the polar curve $r = 2\cos(\theta)$.

Solution: We multiply both sides of $r = 2\cos(\theta)$ by r to get

$$r^2 = 2r\cos(\theta).$$

Now we use the change of coordinates $x = r\cos(\theta)$ and $r^2 = x^2 + y^2$:

$$x^2 + y^2 = r^2 = 2r\cos(\theta) = 2x,$$

so our Cartesian equation is:

$$x^2 - 2x + y^2 = 0.$$

15. Let S be the solid obtained by rotating the region bounded by the circle $x^2 + y^2 = 1$ around the line $x = 2$.

(a) (5 points) Set up the integral that computes the volume of S using the disk/washer method.

Solution:

$$\int_{-1}^{1} \pi((2+\sqrt{1-y^2})^2 + (2-\sqrt{1-y^2})^2)dy$$

(b) (5 points) Set up the integral that computes the volume of S using the cylindrical shells method.

Solution:

$$\int_{-1}^{1} 2\pi(2-x)(2\sqrt{1-x^2})dx$$