MA 114 Quiz 8 October 31, 2019

Answer all questions and show your work. Unsupported answers may receive *no credit*. You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

Name: ______________________ Section: ______________________

1. (4 points) Use the definition of a Taylor series to find the first two nonzero terms of the Taylor series for $\cos^2 x$ centered at $a = 0$.

Solution: If $f(x) = \cos^2 x$ then $f'(x) = 2\cos x(-\sin x) = -2\cos x \sin x$. Differentiating again $f''(x) = -2(\cos x \cos x + (-\sin x)\sin x) = -2(\cos^2 x - \sin^2 x)$. Evaluating at 0 gives $f(0) = 1$, $f'(0) = 0$ and $f''(0) = -2(1) = -2$. The first three terms (and first 2 nonzero terms) of the Taylor series are

$$1 + \frac{0}{1!}(x-0) + \frac{-2}{2!}(x-0)^2 = 1 + 0x - x^2$$

2. (a) (4 points) Find the average value of $f(x) = (x-3)^2$ on $[2, 5]$.

Solution: $\displaystyle\int_2^5 (x-3)^2 dx = \frac{1}{3}(x-3)^3\Big|_2^5 = \frac{1}{3}\left((5-3)^3 - (2-3)^3\right) = \frac{1}{3}(8-(-1)) = 3$. So the average value is $\dfrac{1}{5-2}3 = \dfrac{1}{3}3 = 1$.

(b) (2 points) Find a c so that $f(c)$ is the average value of f.

Solution: $1 = (x-3)^2$ so either $1 = x-3$ or $-1 = x-3$. In the first case $x = 4$ and in the second $x = 2$. Either of these are solutions.