Name: ____________________ Section: ____________________

Answer all questions and show your work. Unsupported answers may receive *no credit.* You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. (4 points) Let $a_n = \sin(n)/\sqrt{n}$. Does the sequence $\{a_n\}$ converge? If the sequence converges, find its limit. You must give a mathematical justification as, for example, by applying a theorem.

Solution: Since
$$-\frac{1}{\sqrt{n}} \leq \frac{\sin(n)}{\sqrt{n}} \leq \frac{1}{\sqrt{n}}$$
and
$$\lim_{n\to\infty}\left(-\frac{1}{\sqrt{n}}\right) = \lim_{n\to\infty}\left(\frac{1}{\sqrt{n}}\right) = 0,$$
the limit $\lim_{n\to\infty} a_n$ exists and is equal to 0 by the Squeeze Theorem.

2. (6 points) For each of the following sequences, determine whether it converges, and if yes, find its limit:

(a) (2 points) $a_n = \dfrac{e^n}{3^{n+1}}$

(b) (2 points) $b_n = \dfrac{n^2+1}{(2n+1)^2}$

(c) (2 points) $c_n = \dfrac{e^n}{n^2}$

Solution: (a) We have
$$\frac{e^n}{3^{n+1}} = \frac{1}{3}\cdot\left(\frac{e}{3}\right)^n = \frac{r^n}{3}$$
with $r = e/3 < 1$. Thus the sequence converges to 0.

(b) We have
$$\frac{n^2+1}{(2n+1)^2} = \frac{n^2+1}{4n^2+4n+1} = \frac{1+1/n^2}{4+4/n+1/n^2}$$
Thus $\{b_n\}$ converges to 1/4.

(c) We have $c_n = f(n)$ with $f(x) = \dfrac{e^x}{x^2}$. Using l'Hôpital's Rule twice,
$$\lim_{x\to\infty}\frac{e^x}{x^2} = \lim_{x\to\infty}\frac{e^x}{2x} = \lim_{x\to\infty}\frac{e^x}{2} = \infty.$$
Thus $\{c_n\}$ diverges to infinity.