Name: ____________________ Section: ____________________

Answer all questions and show your work. Unsupported answers may receive *no credit.* You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. (a) (2 points) State the Limit Comparison Test for series.

   (b) (3 points) Use the Limit Comparison Test to compare the series

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{8^n}{5+7^n}$$

with an appropriate geometric series and determine whether $\sum_{n=1}^{\infty} a_n$ converges or diverges.

Solution: (a) Suppose $\sum a_n$ and $\sum b_n$ are series with positive terms. If $\lim_{n\to\infty} a_n/b_n = c \neq 0$, then either both series converge or both series diverge.

(b) If we let

$$a_n = \frac{8^n}{5+7^n} \quad \text{and} \quad b_n = \left(\frac{8}{7}\right)^n,$$

then

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{7^n}{5+7^n} = \lim_{n\to\infty} \frac{1}{5/7^n+1} = 1 \neq 0.$$

Since $\sum b_n$ diverges (as a geometric series with common ratio $8/7 > 1$), then $\sum a_n$ diverges by the Limit Comparison Test.

2. (5 points) Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^4}$. How many terms of the series would you need to add to find its sum to within $\frac{1}{3000}$?

Solution: Recall the Remainder Estimate for the Integral Test:

$$s - s_N \leq \int_N^{\infty} f(x)\,dx.$$

Here $f(x) = 1/x^4$ and

$$\int_N^{\infty} \frac{dx}{x^4} = \lim_{t\to\infty} \int_N^t \frac{dx}{x^4} = \lim_{t\to\infty} \left[-\frac{1}{3x^3}\right]_N^t = \frac{1}{3N^3}.$$

Solving

$$\frac{1}{3N^3} \leq \frac{1}{3000}$$

for N, gives $N \geq 10$.