Name: _____ Section: _____

Answer all questions and show your work. Unsupported answers may receive *no credit*. You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. Consider a semicircular lamina Q described by

$$Q = \{(x, y) \mid x^2 + y^2 \leq 4,\ y \geq 0,\ -2 \leq x \leq 2\}$$

with density $\rho = 3$ units of mass per unit of area.

(a) (2 points) Use geometry to find the total mass m of the lamina.

Solution: Since the area of Q is 2π, the total mass is $m = 6\pi$.

(b) (2 points) Compute the moment M_x of the lamina about the x-axis.

Solution: We have

$$M_x = \frac{3}{2}\int_{-2}^{2} (\sqrt{4 - x^2})^2\, dx = \frac{3}{2}\int_{-2}^{2} (4 - x^2)\, dx = \frac{3}{2}\left[4x - \frac{x^3}{3}\right]_{x=-2}^{x=2} = 16.$$

(c) (2 points) Find the center of mass of the lamina.

Solution: Clearly $M_y = 0$ by symmetry. Thus the center of mass is

$$(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right) = \left(0, \frac{16}{6\pi}\right) = \left(0, \frac{8}{3\pi}\right).$$

2. (4 points) Consider the curve C with parametric equations

$$x = t^2, \quad y = t^3 - 3t + 4.$$

Is the point $(4, 2)$ on the curve C? If the answer is no, explain why. If the answer is yes, find t.

Solution: Equating the x-coordinates, we have $t^2 = 4$, i.e. $t = \pm 2$. For $t = 2$, $t^3 - 3t + 4 = 6 \neq 2$. For $t = -2$, we have $t^3 - 3t + 4 = 2$. Thus the point $(4, 2)$ is on the curve C and it corresponds to $t = -2$.