Name: _____ Section: _____

Answer all questions and show your work. Unsupported answers may receive *no credit*. You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. (3 points) Evaluate $\int xe^{2x}\,dx$

Solution: We let $u = x$ and $dv = e^{2x}\,dx$. With these choices, $du = dx$ and $v = \frac{1}{2}e^{2x}\,dx$. Integrating by parts gives

$$\begin{aligned}\int xe^{2x}\,dx &= \frac{x}{2}e^{2x} - \frac{1}{2}\int 1e^{2x}\,dx \\ &= \frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + C.\end{aligned}$$

The last line is follows since $\frac{1}{4}e^{2x}$ is an anti-derivative of $\frac{1}{2}e^{2x}$.

Grading: Correct expressions for du and v (1 point). Integrating by parts as in first line of displayed equation (1 point). Final answer (1 point).

Students are encouraged (but not required) to check their answer by differentiation.

2. (3 points) Evaluate $\int x\cos(x^2)\,dx$

Solution: This might appear to be another problem involving integration by parts, but it is best approached by substituting $u = x^2$ and $du = 2x\,dx$. Carrying out the substitution gives:

$$\begin{aligned}\int x\cos(x^2)\,dx = \frac{1}{2}\int \cos(u)\,du \\ &= \frac{1}{2}\sin(u) + C \\ &= \frac{1}{2}\sin(x^2) + C\end{aligned}$$

Grading: Choosing substitution method and setting $u = x^2$ (1 point). Substituting u for x and setting $x\,dx = \frac{1}{2}du$ (1 point). Answer (1 point).

Students are encouraged (but not required) to check their answer by differentiation.

3. (4 points) Let f be a twice differentiable function with $f(3) = 7$, $f(5) = 9$, $f'(3) = 4$ and $f'(5) = 6$. Evaluate

$$\int_3^5 xf''(x)\,dx.$$

Solution: Integrate by parts to obtain

$$\begin{aligned}\int_3^5 xf''(x)\,dx &= xf'(x)|_{x=3}^5 - \int_3^5 f'(x)\,dx \\ &= 5f'(5) - 3f'(3) - \int_3^5 f'(x)\,dx.\end{aligned}$$

Use that f is an anti-derivative of f' and the Fundamental Theorem of Calculus to obtain

$$\int_3^5 f'(x)\,dx = f(5) - f(3).$$

Substituting values gives

$$\int_3^5 xf''(x)\,dx = 5f'(5) - 3f'(3) - (f(5) - f(3)) \quad = 5 \cdot 6 - 3 \cdot 4 - (9 - 7) = 16.$$

Grading: (2 points for integrating by parts), (1 point for using Fundamental theorem of Calculus), (1 point for evaluating answer).

This is similar to a problem on Worksheet #1.