Name: ______________________ Section: ______________________

Answer all questions and show your work. Unsupported answers may receive *no credit*. You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. (5 points) Evaluate the integral $\displaystyle\int_0^{\pi/2} \cos^3(x)\,dx$.

Solution: We use the identity, $\cos^2(x) = 1 - \sin^2(x)$ to rewrite the integral as

$$\int_0^{\pi/2} \cos^3(x)\,dx = \int_0^{\pi/2} (1 - \sin^2(x))\cos(x)\,dx.$$

Making the substitution $u = \sin(x)$, $x = 0$ corresponds to $u = 0$, $x = \pi/2$ corresponds to $u = 1$, and $du = \cos(x)\,dx$ we obtain

$$\int_0^{\pi/2} (1 - \sin^2(x))\cos(x)\,dx = \int_0^1 (1 - u^2)\,du = (u - \frac{u^3}{3})\Big|_{u=0}^{1} = 2/3.$$

Trig identity (1 point), substitute $u = \sin(x)$, $du = \cos(x)\,dx$ (1 point), change limits (1 point), integrate polynomial (1 point), answer (1 point).

2. (5 points) Evaluate the anti-derivative $\displaystyle\int \sqrt{4 - x^2}\,dx$. Simplify your answer so that it does not contain any trigonometric functions. (Your answer may contain inverse trigonometric functions.)

Solution: We make the substitution $x = 2\sin(u)$ and then $dx = 2\cos(u)\,du$. This gives

$$\int \sqrt{4 - x^2}\,dx = 2\int \sqrt{4 - 4\sin^2(u)}\,\cos(u)\,du = 4\int \cos^2(u)\,du.$$

Using the double angle formula, we have $\cos^2(u) = \dfrac{1 + \cos(2u)}{2}$ and then integrating gives

$$4\int \frac{1}{2} + \frac{1}{2}\cos(2u)\,du = 2u + \sin(2u) + C.$$

Finally, writing $\sin(2u) = 2\sin(u)\cos(u)$, $2\sin(u) = x$ and $2\cos(u) = \sqrt{4 - x^2}$ we may express the answer as

$$2u + \sin(2u) + C = 2\sin^{-1}(x/2) + \frac{x}{2}\sqrt{4 - x^2} + C = 2\sin^{-1}(x/2) + \frac{1}{2}x\sqrt{4 - x^2} + C.$$

Substitute $x = 2\sin(u)$, $dx = 2\cos(u)$ (1 point), simplifying $\sqrt{4-x^2} = 2\cos(u)$ (1 point), identity for $\cos^2(u)$ (1 point), anti-derivative of $2(1+\cos(2u))$ (1 point), simplifying result (1 point).