Name: ______________________________ Section: ________________________

Answer all questions and show your work. Unsupported answers may receive *no credit*. You may not use a calculator on this quiz. Allow 15 minutes for the quiz.

1. (a) (2 points) Find an improper integral so that $\sum_{k=N}^{\infty} \frac{1}{k^2} \leq \int_A^\infty f(x)\,dx$.

   (b) (3 points) Use your answer to part a) to find N so $\sum_{k=N}^{\infty} \frac{1}{k^2} \leq \frac{1}{300}$.

Solution: a) We have $1/k^2 \leq 1/x^2$ if $k-1 \leq x \leq k$. Thus we have

$$\sum_{k=N}^{\infty} \frac{1}{k^2} \leq \int_{N-1}^{\infty} \frac{1}{x^2}.$$

b) Evaluating the integral we have

$$\int_{N-1}^{\infty} \frac{1}{x^2}\,dx = \lim_{R\to\infty} \int_{N-1}^{R} \frac{1}{x^2}\,dx = \lim_{R\to\infty} -x^{-1}\,\Big|_{x=N-1}^{R} = \frac{1}{N-1}.$$

We need $1/(N-1) \leq 1/300$ or $N \geq 301$.

Grading: a) 1 point for integrand $f(x) = 1/x^2$, 1 point for limit $A = N-1$. b) 1 point for anti-derivative, 1 point for value of integral, 1 point for value for N. Give credit for part b) if a wrong answer from part a) is used correctly in part b).

Compare WeBWorK B3 #10.

2. (a) (2 points) For which p does the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converge?

   (b) (3 points) Use the limit comparison test to determine if the series $\sum_{n=1}^{\infty} \frac{2n^2+3n+1}{n^5+5n^3+2}$ is convergent. Your answer should give the series you use for comparison.

Solution: a) The p-series converges for $p > 1$.

b) If we let $a_n = \frac{2n^2+3n+1}{n^5+5n^3+2}$ we can compare with $b_n = 1/n^3$.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^3 \cdot (2n^2+3n+1)}{n^5+5n^3+2} = 2.$$

Since the limit is non-zero and finite and the series b_n converges, the series $\sum_{n=1}^{\infty} a_n$ will converge also.

Grading: a) 2 points for correct answer, b) 1 point for choosing $b_n = 1/n^3$, 1 point for taking limit, 1 point for answer.

Compare WeBWorK B4 #8.