`Chapter 10: Practice/review problems`

The collection of problems listed below comprises questions taken from previous MA123 exams.

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[1]. Let $h(x) = \int_4^x \sqrt{t^2+3}\,dt$. Find $h'(x)$.

(a) $\frac{1}{2}(x^2+3)^{-1/2}\cdot 2x$ **(b)** $1+\frac{1}{x^2}$ **(c)** $-\frac{2}{x^3}$

(d) $\frac{x^2}{x^2+1}$ **(e)** $\sqrt{x^2+3}$

[2]. Let $F(x) = \int_1^x (2t^2-3t+1)\,dt$. Find $F'(3)$.

(a) 7 **(b)** 8 **(c)** 9 **(d)** 10 **(e)** 11

[3]. If

$$F(x) = \int_2^x (t^2+4t)\,dt,$$

find $F'(3)$.

(a) $\frac{49}{3}$ **(b)** 21 **(c)** $\frac{64}{3}$ **(d)** 27 **(e)** 36

[4]. Let $A(x) = \int_0^x (t^2+t^4+t^6)\,dt$. Find the value of x on $[1,50]$ where $A(x)$ takes its minimum value.

(a) 1 **(b)** $1^2+1^4+1^6$ **(c)** 25

(d) $50^2+50^4+50^6$ **(e)** 50

[5]. Find

$$\int_0^{12} (s^2+3s+1)\,ds$$

(a) 181 **(b)** 804 **(c)** 132 **(d)** 55 **(e)** 1

[6]. Find $\int_1^2 (x^2+2x+1)\,dx$.

(a) 4 **(b)** 19/3 **(c)** 9 **(d)** 3 **(e)** 29/3

[7]. Evaluate the limit

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n} f(k/n)$$

where $f(x) = x^2$.

(**Hint:** Draw a picture and relate the limit to an integral.)

(a) 1/5 **(b)** 1/4 **(c)** 1/3 **(d)** 1/2 **(e)** 1

[8]. Let $f(x) = \begin{cases} x & \text{if } 0 \le x < 1 \\ 2 & \text{if } 1 \le x < 4 \end{cases}$. Evaluate the integral $\displaystyle\int_0^3 f(x)\,dx$.

(a) 7/2 **(b)** 9/2 **(c)** 11/2 **(d)** 13/2 **(e)** 15/2

[9]. Suppose $\displaystyle\int_0^1 f(x)\,dx = 4$ and $\displaystyle\int_0^1 g(x)\,dx = 5$. What is the value of $\displaystyle\int_0^1 [2f(x) + g(x)]\,dx$?

(a) 19 **(b)** 17 **(c)** 15 **(d)** 13 **(e)** 11

[10]. Use the Fundamental Theorem of Calculus to compute $\displaystyle\int_1^6 \sqrt{x+3}\,dx$.

(a) 37/3 **(b)** 38/3 **(c)** 39/3 **(d)** 40/3 **(e)** 41/3

[11]. Find the general antiderivative $\displaystyle\int (x+5)^2\,dx$.

(a) $3(x+5)^2 + C$ **(b)** $(x+2)^{-1} + C$ **(c)** $-2(x+2)^{-3} + C$

(d) $-(x+2)^{-1} + C$ **(e)** $\dfrac{1}{3}(x+5)^3 + C$

[12]. Find

$$\int \frac{x^3+1}{x^2}\,dx$$

(a) $\dfrac{(x^4/4)+x}{(x^3/3)} + C$ **(b)** $\dfrac{x^3+2}{2x} + C$ **(c)** $\dfrac{(x^4/4)-x}{(x^3/3)} + C$

(d) $\dfrac{x^3-2}{2x} + C$ **(e)** $\dfrac{x^4+x}{x^3} + C$

[13]. What is the average of the function $h(t) = t^2 + 1$ on the interval $[1, 4]$? Recall that the average of $f(t)$ on an interval $[a, b]$ equals the constant value A such that the area under the graph of the constant function A equals the area under the graph of $f(t)$ for the interval $[a, b]$. In other words,

$$\int_a^b f(t)\,dt = \int_a^b A\,dt.$$

(a) 8 **(b)** 10 **(c)** 12 **(d)** 14 **(e)** 16

[14]. What is the average of the function $h(t) = t^3 + 1$ on the interval $[1, 4]$?

(a) $\dfrac{247}{12}$ **(b)** $\dfrac{257}{12}$ **(c)** $\dfrac{267}{12}$ **(d)** $\dfrac{277}{12}$ **(e)** $\dfrac{279}{12}$