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GOOD LUCK!

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For grading use:

Multiple Choice	Short Answer
(number right) (5 points each)	(out of 10 points)

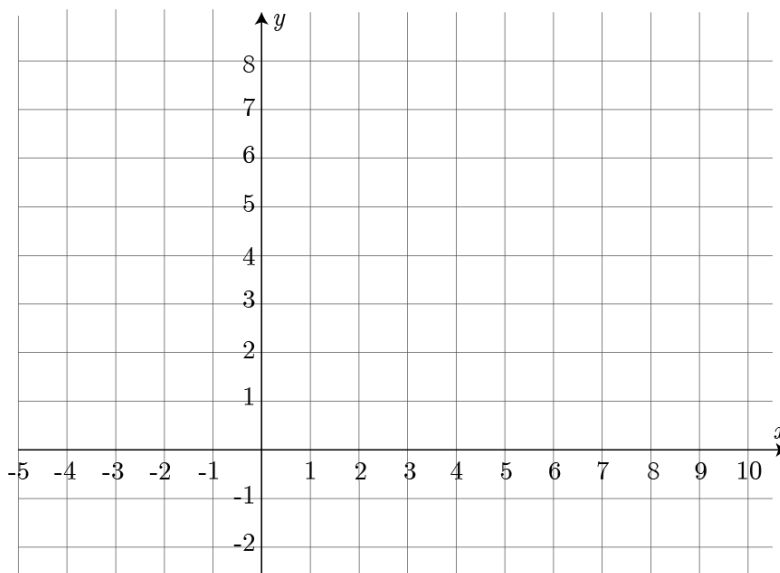
Total
(out of 100 points)

Spring 2015 Exam 3 Short Answer Questions

Write your answers on this page.

1. Sketch the graph of a **continuous** function $y = f(x)$ which satisfies the following:

$$f(8) = 2, \quad f'(x) \leq 0 \text{ for all } x; \quad f''(x) > 0 \text{ for } x < 3; \quad f''(x) < 0 \text{ for } x > 3.$$



2. The area of a square is increasing at a rate of $140 \text{ cm}^2/\text{min}$. At what rate is the length of the side of the square increasing when the area is 25 cm^2 ? *You must show proper, appropriate, and legible work to be sure you will get full credit.*

Final answer: _____

Name: _____

Multiple Choice Questions

*Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.*

3. Where is the function $f(t) = t^3 - 9t^2 - 48t + 1$ decreasing?

Possibilities:

- (a) $-2 < t < 8$
- (b) $t < 3$
- (c) $t < -2$ and $t > 8$
- (d) $f(t)$ is always decreasing
- (e) $t > 3$

4. Where is the function $f(t) = t^3 - 9t^2 - 48t + 1$ concave up?

Possibilities:

- (a) $f(t)$ is always concave up
 - (b) $t > 3$
 - (c) $-2 < t < 8$
 - (d) $t < 3$
 - (e) $t < -2$ and $t > 8$
-

5. If $g'(t) = 4 - t^2$, where is the function $g(t)$ decreasing?

Possibilities:

- (a) $t < 0$
- (b) $-2 < t < 2$
- (c) $t < -2$ and $t > 2$
- (d) $f(t)$ is always decreasing
- (e) $t > 0$

6. If $g'(t) = 4 - t^2$, where is the function $g(t)$ concave up?

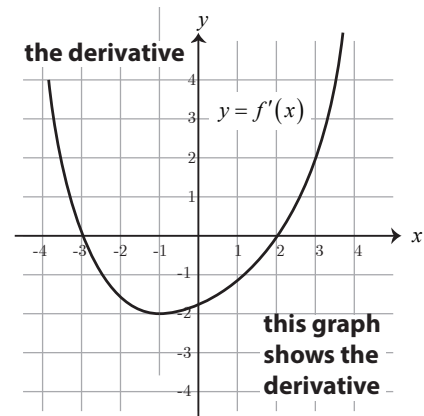
Possibilities:

- (a) $t < 0$
- (b) $-2 < t < 2$
- (c) $t < -2$ and $t > 2$
- (d) $f(t)$ is always concave up
- (e) $t > 0$

-
7. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$.
Where is the regular function $f(x)$ decreasing?

Possibilities:

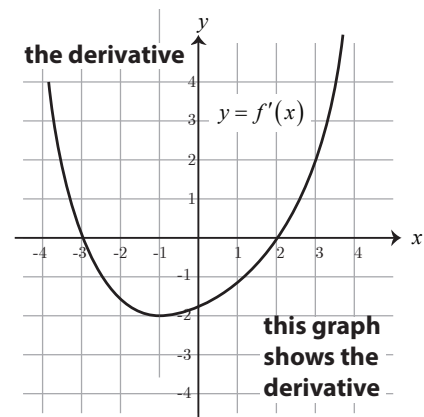
- (a) $(-2, \infty)$
- (b) $(-\infty, -1)$
- (c) $(-1, \infty)$
- (d) $(-3, 2)$
- (e) $(-\infty, -3)$ and $(2, \infty)$



-
8. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$.
Where is the regular function $f(x)$ concave up?

Possibilities:

- (a) $(-2, \infty)$
- (b) $(-\infty, -3)$ and $(2, \infty)$
- (c) $(-3, 2)$
- (d) $(-\infty, -1)$
- (e) $(-1, \infty)$



-
9. Two trains leave the same station at different times, one traveling due East, and the other traveling due North. At 2pm the eastbound train is traveling at 50 mph and is 400 miles from the station, while the northbound train is traveling at 60 mph and is 300 miles from the station. At what rate is the distance between the trains increasing?

Possibilities:

- (a) 76000 mph
- (b) $10\sqrt{61}$ mph
- (c) 500 mph
- (d) 76 mph
- (e) 110 mph

-
10. Find the area of the largest rectangle whose sides are parallel to the coordinate axes, whose bottom-left corner is at $(0, 0)$ and whose top-right corner is on the graph of $y = 6x - x^2$.

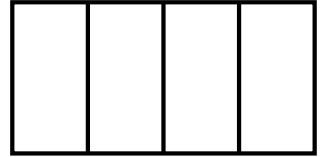
Possibilities:

- (a) 27
- (b) 3
- (c) 30
- (d) 32
- (e) 0

-
11. A farmer builds a rectangular pen with 4 vertical partitions (5 vertical sides) using 600 feet of fencing. What is the maximum possible total area of the pen?

Possibilities:

- (a) 300
- (b) 7500
- (c) 9000
- (d) 600
- (e) 22500



-
12. A farmer currently has harvested 130 bushels of collard greens that are currently worth \$12.74 per bushel. The way things are going, he expects to be harvesting 3.00 bushels per day, and expects the price to be increasing at \$0.75 per bushel per day. What is the instantaneous rate of change (measured in dollars per day) of the total value of his collard greens?

Possibilities:

- (a) \$135.72 per day
- (b) \$135.73 per day
- (c) \$135.74 per day
- (d) \$135.75 per day
- (e) \$135.76 per day

-
13. Estimate the area under the graph of $-x^2 + 20x$ for x between 4 and 10, by using a partition that consists of 3 equal subintervals of $[4, 10]$ and use the right endpoint of each subinterval as a sample point.

Possibilities:

- (a) 528
- (b) 688
- (c) 560
- (d) 280
- (e) 488

-
14. A train travels in a straight westward direction along a track. The speed of the train varies, but it is measured at regular time intervals of $1/10$ hour. The measurements for the first half hour are:

time	0	.1	.2	.3	.4	.5
speed	0	4	10	14	20	23

Estimate the total distance (in miles) traveled by the train during the first half hour by assuming the speed is a linear function of t on the subintervals. The speed in the table is given in miles per hour. Use all six speed measurements in your estimate.

Possibilities:

- (a) 7.00 miles
- (b) 7.10 miles
- (c) 2.00 miles
- (d) 11.50 miles
- (e) 5.95 miles

-
15. One way to approximate $\int_A^{59} e^{19-2x} dx$ is with the sum $\sum_{k=1}^{200} ((\Delta x) \cdot (e^{19-2(9+k\Delta x)}))$ where $\Delta x = \frac{1}{4}$. What is the best value of A to use?

Possibilities:

- (a) 9
- (b) $\frac{1}{4}$
- (c) 1.359140914
- (d) 0.01
- (e) 200

-
16. Suppose you estimate the area under the graph of $f(x) = x^3$ from $x = 5$ to $x = 25$ by adding the areas of the rectangles as follows: partition the interval into 20 equal subintervals and use the right endpoint of each interval to determine the height of the rectangle. What is the area of the 7th rectangle?

Possibilities:

- (a) 25
 - (b) 1728
 - (c) 105400
 - (d) $\frac{6095}{4}$
 - (e) 1331
-

17. Evaluate the difference of sums

$$\left(\sum_{k=1}^{40000} (6k^3 + 5) \right) - \left(\sum_{k=3}^{40000} (6k^3 + 5) \right)$$

Possibilities:

- (a) 800020000
- (b) 3840000000000005
- (c) 64
- (d) 0
- (e) ∞

18. Evaluate the sum

$$\sum_{k=1}^N (11k^2)$$

Possibilities:

- (a) $11N^2$
- (b) $11N^2 - 11$
- (c) $11 \frac{N(N+1)}{2}$
- (d) $11N^2 + 11$
- (e) $11 \frac{N(N+1)(2N+1)}{6}$

19. Evaluate the sum $5 + 10 + 15 + 20 + 25 + 30 + 35 + 40 + 45 + 50 + \cdots + 370 + 375$.

Possibilities:

- (a) 14250
- (b) 70500
- (c) 717250
- (d) 140625
- (e) 1020

20. Evaluate the sum $\frac{1}{13} + \frac{4}{13} + \frac{9}{13} + \frac{16}{13} + \frac{25}{13} + \frac{36}{13} + \frac{49}{13} + \frac{64}{13} + \frac{81}{13} + \frac{100}{13} + \cdots + \frac{841}{13} + \frac{900}{13}$.

Possibilities:

- (a) $\frac{13515}{13}$
- (b) $\frac{810000}{169}$
- (c) $\frac{2126}{13}$
- (d) $\frac{9455}{13}$
- (e) $\frac{410850}{169}$

Some Formulas

1. Summation formulas:

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

2. Areas:

(a) Triangle $A = \frac{bh}{2}$

(b) Circle $A = \pi r^2$

(c) Rectangle $A = lw$

(d) Trapezoid $A = \frac{h_1 + h_2}{2} b$

3. Volumes:

(a) Rectangular Solid $V = lwh$

(b) Sphere $V = \frac{4}{3}\pi r^3$

(c) Cylinder $V = \pi r^2 h$

(d) Cone $V = \frac{1}{3}\pi r^2 h$

4. Distance:

(a) Distance between (x_1, y_1) and (x_2, y_2)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

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