MA123 - Elem. Calculus Final Exam

Fall 2015
2015-12-17

Name: $\qquad$ Sec.: $\qquad$

Do not remove this answer page - you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.
The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write
(a) b c de

You have two hours to do this exam. Please write your name on this page, and at the top of page three.

## GOOD LUCK!

3. (a) b c d e
4. (a) b c d e
5. (a) b c d e
6. (a) b c d (e)
7. (a) b c d e
8. (a) b c d e
9. (a) b d e
10. (a) b c d e
11. (a) b e e
12. (a) b c d e
13. (a) b c d e
14. (a) b c d e
15. (a) b c d e
16. (a) b c d e
17. (a) b c d e
18. (a) b c d e
19. (a) b c d e
20. (a) b c d e

## For grading use:

| Multiple Choice | Short Answer |
| :---: | :---: |
|  |  |
| (number right) $\quad$ (5 points each) | (out of 10 points) |


| Total |  |
| :--- | :--- |
|  | (out of 100 points) |

## Fall 2015 Exam 4 Short Answer Questions

Write answers on this page. You must show appropriate legible work to be sure you will get full credit.

1. Evaluate $\int_{0}^{T} \sqrt{3 x+10} d x$. Show steps clearly andcircle your answer. You do NOT need to simplify your final answer.
2. If the derivative of a function $g(x)$ is given by $g^{\prime}(x)=(x+2)(x+5)$, find interval(s) where $g(x)$ is concave up. Show work clearly.
$\qquad$
$\qquad$

## Multiple Choice Questions

Show all your work on the page where the question appears. Clearly mark your answer both on the cover page on this exam and in the corresponding questions that follow.
3. Find the limit as $n$ tends to infinity. Here $C$ is a fixed real number.

$$
\lim _{n \rightarrow \infty} \frac{(C n+1)^{2}}{4 n^{2}+3 n+7}
$$

## Possibilities:

(a) $\frac{1}{4} C^{2}$
(b) $\infty$
(c) $\frac{1}{16} C^{2}$
(d) $\frac{1}{14} C$
(e) 0
4. Evaluate the limit as $n$ tends to infinity. Note: you will have to use some of the summation formulas (see formula sheet on backpage) to simplify.

$$
\lim _{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^{n} \frac{6 k^{2}}{n^{2}}
$$

## Possibilities:

(a) 2
(b) 1
(c) 3
(d) 4
(e) 5
5. Assuming $x>0$, evaluate the definite integral

$$
\int_{7}^{x} \frac{6}{t} \mathrm{~d} t
$$

## Possibilities:

(a) $-\frac{6}{x^{2}}+\frac{6}{49}$
(b) $12 \sqrt{x}-12 \sqrt{7}$
(c) $6 \sqrt{x}$
(d) $6 \ln (|x|)-6 \ln (7)$
(e) $\frac{6}{\frac{1}{2} x^{2}}-\frac{12}{49}$
6. Use the Fundamental Theorem of Calculus to compute the derivative, $F^{\prime}(x)$, of $F(x)$, if

$$
F(x)=\int_{1}^{x+9}\left(t^{2}+7 t+3\right) \mathrm{d} t
$$

## Possibilities:

(a) $2 x+7$
(b) $\frac{1}{3} x^{3}+\frac{7}{2} x^{2}+3 x-\left(\frac{1}{3} 1^{3}+\frac{7}{2} 1^{2}+3(1)\right)$
(c) $x^{2}+7 x+3$
(d) $\frac{1}{3}(x+9)^{3}+\frac{7}{2}(x+9)^{2}+3(x+9)-\left(\frac{1}{3} 1^{3}+\frac{7}{2} 1^{2}+3(1)\right)$
(e) $(x+9)^{2}+7(x+9)+3$
7. Find the value of $x$ at which

$$
F(x)=\int_{3}^{x}\left(-t^{4}-t^{2}-7\right) \mathrm{d} t
$$

takes its minimum value on the interval $[9,800]$.
Possibilities:
(a) 3
(b) 97
(c) $\frac{393}{5}$
(d) 9
(e) 800
8. Evaluate the integral

$$
\int_{0}^{x}(7 t+9)^{15} \mathrm{~d} t
$$

## Possibilities:

(a) $\frac{1}{7(16)}(7 x+9)^{16}-\frac{9^{16}}{7(16)}$
(b) $\frac{1}{15}(7 x+9)^{15}-\frac{9^{15}}{15}$
(c) $\frac{1}{16}(7 x+9)^{16}-\frac{9^{16}}{16}$
(d) $16(7 x+9)^{16}-15 \cdot 9^{16}$
(e) $\frac{1}{16} x^{16}-\frac{9^{16}}{16}$
9. Suppose a rock is dropped from a lunar cliff. After $t$ seconds, its speed in feet per second is $v(t)=\frac{53}{10} t$, at least until it lands. If the rock lands after 9 seconds, how high (in feet) is the cliff?

Possibilities:
(a) $\frac{9}{2}$ feet
(b) $\frac{53}{90}$ feet
(c) 9 feet
(d) $\frac{4293}{20}$ feet
(e) $\frac{477}{10}$ feet
10. Compute $\lim _{t \rightarrow 2} \frac{t^{2}+6 t-16}{t^{2}+2 t-8}$

## Possibilities:

(a) $\frac{4}{3}$
(b) $\frac{5}{3}$
(c) 2
(d) $\frac{7}{3}$
(e) The limit does not exist.
11. Find the average rate of change of $f(x)=\sqrt{x}$ from $x=49$ to $x=81$.

## Possibilities:

(a) $\frac{\log (49)+\log (81)}{2}$
(b) $\frac{1}{2}(81)^{-1 / 2}-\frac{1}{2}(49)^{-1 / 2}$
(c) $\frac{\sqrt{81}-\sqrt{49}}{81-49}$
(d) $\frac{\sqrt{81}-\sqrt{49}}{\sqrt{49}-\sqrt{81}}$
(e) $\frac{1}{49}-\frac{1}{81}$
12. If an amount of $x$ dollars is invested at $5 \%$ interest compounded continuously, and at the end of 2 years the value of the investment is $\$ 6000$, find $x$.

## Possibilities:

(a) $\$ 2853.69$
(b) $\$ 600$
(c) $\$ 5000$
(d) $\$ 5429.02$
(e) $\$ 6631.02$
13. The tangent line to the graph of $f$ at $x=7$ has equation $y=9(x-7)+3$. Find $f(7)$ and $f^{\prime}(7)$.

## Possibilities:

(a) $f(3)=7, \quad f^{\prime}(3)=9$
(b) $f(7)=9, \quad f^{\prime}(7)=3$
(c) $f(7)=3, \quad f^{\prime}(7)=9$
(d) $f(3)=9, \quad f^{\prime}(3)=7$
(e) $f(9)=3, \quad f^{\prime}(9)=7$
14. The graph of $y=f(x)$ is shown below. The function is differentiable, except at $x=$

## Possibilities:

(a) $\mathrm{x}=1, \mathrm{x}=3$, and $\mathrm{x}=4$
(b) $x=1$ only
(c) $\mathrm{x}=1, \mathrm{x}=3, \mathrm{x}=4$, and $\mathrm{x}=6$
(d) $x=4$ only
(e) $x=1$ and $x=4$

15. If $f(x)=8 x^{5}+4 x$ then find the second derivative $f^{\prime \prime}(x)$ :

## Possibilities:

(a) $160 x^{3}$
(b) $40 x^{4}+4$
(c) $40 x^{4}+80 x^{3}+80 x^{2}+40 x+12$
(d) $200 x^{5}$
(e) $160 x^{3}+80 x$
16. Suppose $F(x)=g(x) \cdot h(x+2)$. If $g(0)=8, g^{\prime}(0)=4, h(0)=9, h^{\prime}(0)=7, h(2)=5$, and $h^{\prime}(2)=3$, find $F^{\prime}(0)$.

## Possibilities:

(a) 95
(b) 68
(c) 44
(d) 158
(e) 36
17. Find the derivative, $f^{\prime}(x)$, if $f(x)=e^{x^{3}+3 x^{2}+4 x}$.

## Possibilities:

(a) $e^{3 x^{2}+6 x+4}$
(b) $\left(3 x^{2}+6 x+4\right) e^{x^{3}+3 x^{2}+4 x}$
(c) $\ln \left(x^{3}+3 x^{2}+4 x\right)$
(d) $\left(3 x^{2}+6 x+4\right) e^{x}$
(e) $\frac{3 x^{2}+6 x+4}{x^{3}+3 x^{2}+4 x}$
18. Suppose the derivative of $g(t)$ is $g^{\prime}(t)=9(t-7)(t-3)(t-8)$. For $t$ in which interval(s) is $g$ increasing?

## Possibilities:

(a) $\left(-\infty, 6-\frac{1}{3} \sqrt{21}\right) \cup\left(6+\frac{1}{3} \sqrt{21}, \infty\right)$
(b) $(9,3) \cup(7,8)$
(c) $\left(6-\frac{1}{3} \sqrt{21}, 6+\frac{1}{3} \sqrt{21}\right)$
(d) $(-\infty, 3) \cup(7,8)$
(e) $(3,7) \cup(8, \infty)$
19. Find the area of the largest rectangle whose sides are parallel to the coordinate axes, whose bottomleft corner is at $(0,0)$ and whose top-right corner is on the graph of $y=6 x-x^{2}$.

Possibilities:
(a) 0
(b) 32
(c) 30
(d) 27
(e) 3
20. A farmer currently has harvested 290 bushels of kale that are currently worth $\$ 10.51$ per bushel. The way things are going, he expects to be harvesting 3.00 bushels per day, and expects the price to be increasing at $\$ 0.25$ per bushel per day. What is the instantaneous rate of change (measured in dollars per day) of the total value of his kale?

## Possibilities:

(a) $\$ 104.03$ per day
(b) $\$ 104.04$ per day
(c) $\$ 104.05$ per day
(d) $\$ 104.06$ per day
(e) $\$ 104.07$ per day

## Some Formulas

## 1. Summation formulas:

$$
\begin{gathered}
\sum_{k=1}^{n} k=\frac{n(n+1)}{2} \\
\sum_{k=1}^{n} k^{2}=\frac{n(n+1)(2 n+1)}{6}
\end{gathered}
$$

2. Areas:
(a) Triangle $\quad A=\frac{b h}{2}$
(b) Circle $A=\pi r^{2}$
(c) Rectangle $A=l w$
(d) Trapezoid $A=\frac{h_{1}+h_{2}}{2} b$

## 3. Volumes:

(a) Rectangular Solid $\quad V=l w h$
(b) Sphere $\quad V=\frac{4}{3} \pi r^{3}$
(c) Cylinder $\quad V=\pi r^{2} h$
(d) Cone $\quad V=\frac{1}{3} \pi r^{2} h$

## 4. Distance:

(a) Distance between $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$

$$
D=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}
$$

MA123 - Elem. Calculus Final Exam

Fall 2015
2015-12-17

Name: $\qquad$ Sec.: $\qquad$

Do not remove this answer page - you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.
The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write
(a) b c de

You have two hours to do this exam. Please write your name on this page, and at the top of page three.

## GOOD LUCK!

3. (a) b c d e
4. (a) b c d e
5. (a) b c d e
6. (a) b c d (e)
7. (a) b c d e
8. (a) b c d e
9. (a) b c d e
10. (a) b c d e
11. (a) b c d e
12. (a) b c d e
13. (a) b c d e
14. (a) b c d e
15. (a) b c d e
16. (a) b c d e
17. (a) b c d e
18. (a) b c d e
19. (a) b c d e
20. (a) b c d e

## For grading use:

| Multiple Choice | Short Answer |
| :---: | :---: |
| (number right) (5 points each) | (out of 10 points) |


| Total |  |
| :--- | :--- |
|  | (out of 100 points) |

