MA 123, Chapter 3:

The idea of limits (pp. 47-67, Gootman)

Chapter's Goal:

- Evaluate limits.
- Evaluate one-sided limits.
- Understand the concepts of continuity and differentiability and their relationship.

Example 1 (a):

Use the tables to help evaluate $\lim_{x \to 2} \dfrac{x^2+8}{x+2}$.

x gets close to 2 from the left				
x	1.8	1.9	1.99	1.999
$\dfrac{x^2+8}{x+2}$				

Example 1 (a) (continued):

x gets close to 2 from the right				
2.001	2.01	2.1	2.2	x
				$\dfrac{x^2+8}{x+2}$

Example 1 (b):

Suppose that, instead of calculating all the values in the above tables, you simply substitute the value $x = 2$ into $\dfrac{x^2+8}{x+2}$. What do you find?

Example 2:

Compute $\lim_{x \to 1} \left((x^2 + 4x + 3) \cdot (2x - 4) \right)$.

Example 3:

Compute $$\lim_{x\to 1} \frac{x^2 - 2x + 1}{x + 1}.$$

Example 4:

Suppose $\lim_{x \to 3} f(x) = -2$ and $\lim_{x \to 3} g(x) = 4$. Determine

$$\lim_{x \to 3} \left((x+1) \cdot f(x)^2 + \frac{x+2}{g(x)} \right).$$

Example 5:

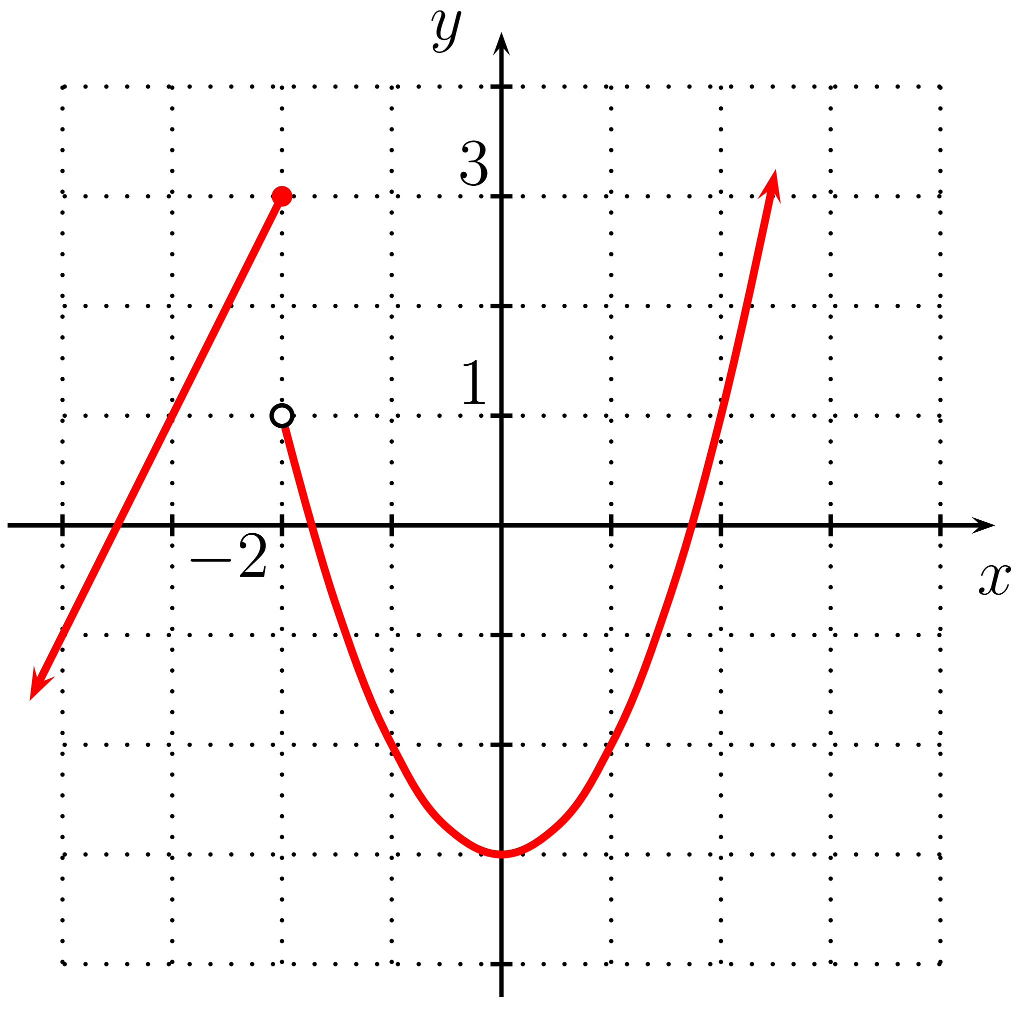
The graph of the function

$$h(x) = \begin{cases} x^2 - 3, & \text{if } x > -2; \\ 2x + 7, & \text{if } x \leq -2 \end{cases}$$

is shown on the next slide.

Analyze $\lim_{x \to -2} h(x)$.

Example 5 (continued):



Example 6:

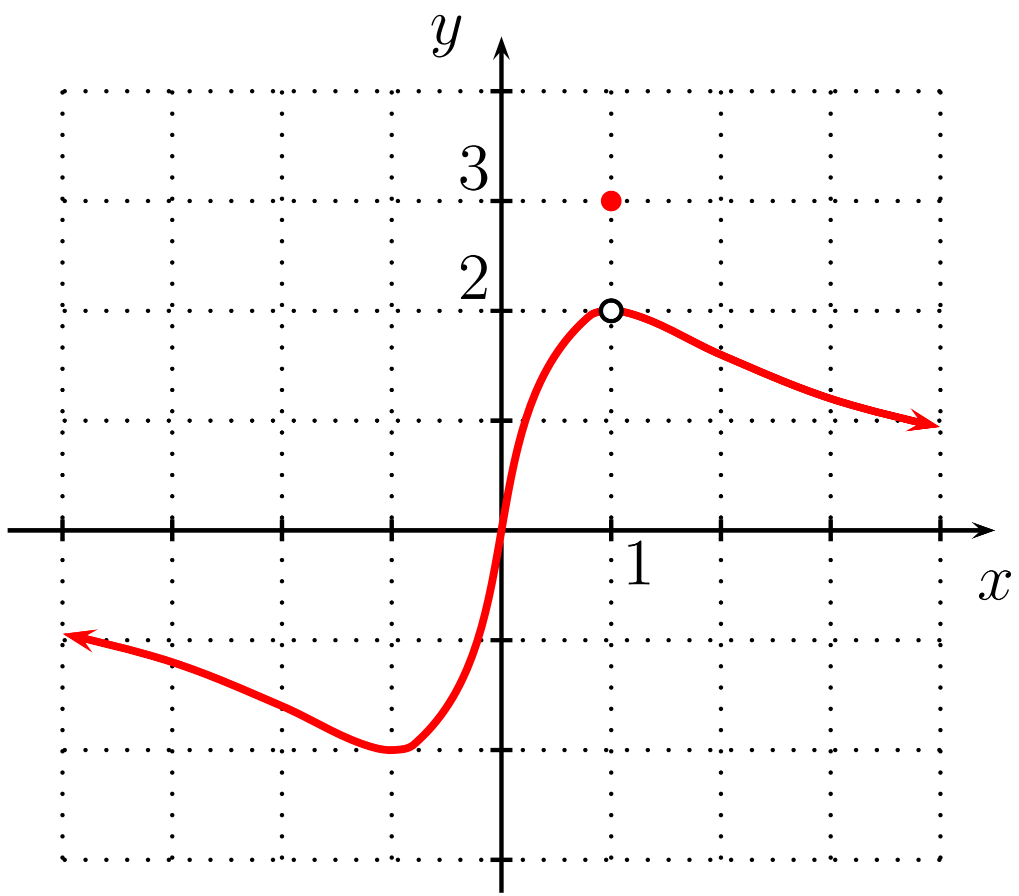
The graph of the function

$$g(x) = \begin{cases} \dfrac{4x}{x^2+1}, & \text{if } x \neq 1; \\ 3, & \text{if } x = 1. \end{cases}$$

is shown on the next slide.

Compute $\lim_{x \to 1} g(x)$.

Example 6 (continued):



Example 7:

Analyze $\lim_{x \to 1} \frac{5}{(x-1)^2}$.

Example 8:

Analyze $\displaystyle \lim_{x\to 1} \frac{2}{x-1}.$

Example 9:

Analyze the limit $\lim_{x \to 0} \frac{2}{\sqrt{x}}$

Example 10:

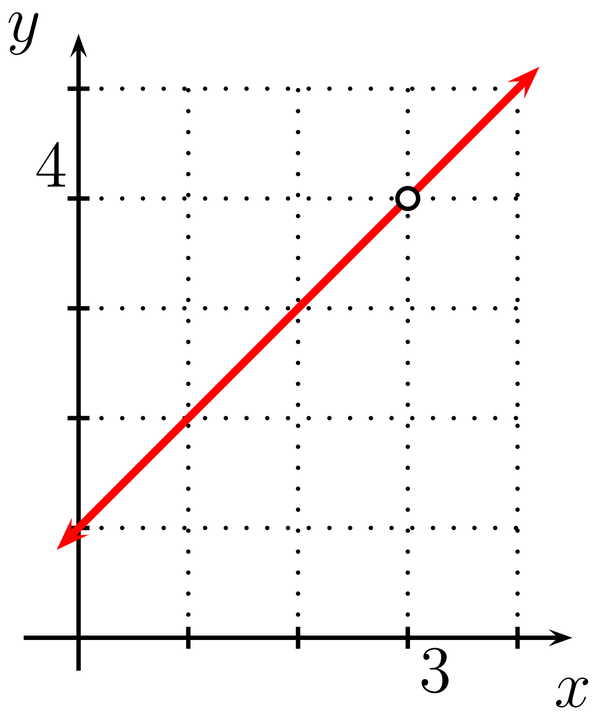
Find the limit $\lim_{x \to 0} \frac{4x}{x}$.

Example 11:

Find the limit $\lim_{x\to 0}\left(\frac{2}{x}+\frac{5x-2}{x}\right).$

Example 12:

Find the limit $\lim_{x\to 3} \frac{x^2 - 2x - 3}{x - 3}$.



Example 13:

Find the limit $\lim_{h \to 0} \frac{(h-3)^2 - 9}{h}$.

Example 14:

Find the limits

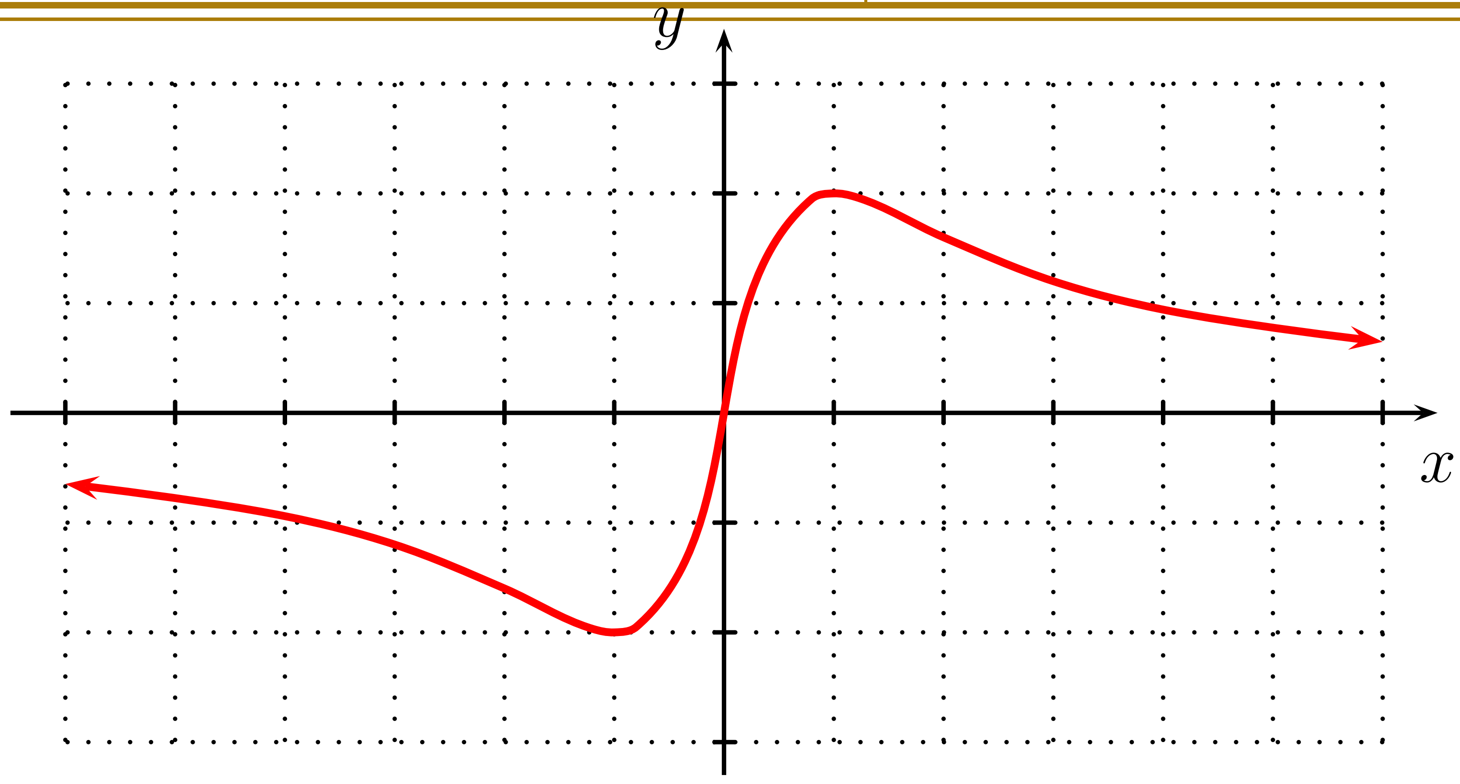
$$\lim_{x\to 2^+} \frac{|3x-6|}{x-2} \qquad \lim_{x\to 2^-} \frac{|3x-6|}{x-2} \qquad \lim_{x\to 2} \frac{|3x-6|}{x-2}.$$

Example 15:

Let $p(x) = \dfrac{4x}{x^2+1}$. Find the limits

$$\lim_{x\to+\infty} \frac{4x}{x^2+1} \qquad \lim_{x\to-\infty} \frac{4x}{x^2+1}.$$

Example 15 (continued):



Example 16:

Find the limit $$\lim_{x\to\infty} \frac{(2x+1)^2}{5x^2+2x+1}.$$

Example 17:

Find the limit $$\lim_{x\to\infty} \frac{(3x+2)^3(5x+1)\sqrt{4x^6+1}}{(x+1)(2x+3)^2(4x+5)^3}$$

Example 18:

Consider the function

$$f(x) = \begin{cases} x^2 - 3, & \text{if } x \leq 1; \\ 2x + B, & \text{if } x > 1 \end{cases}$$

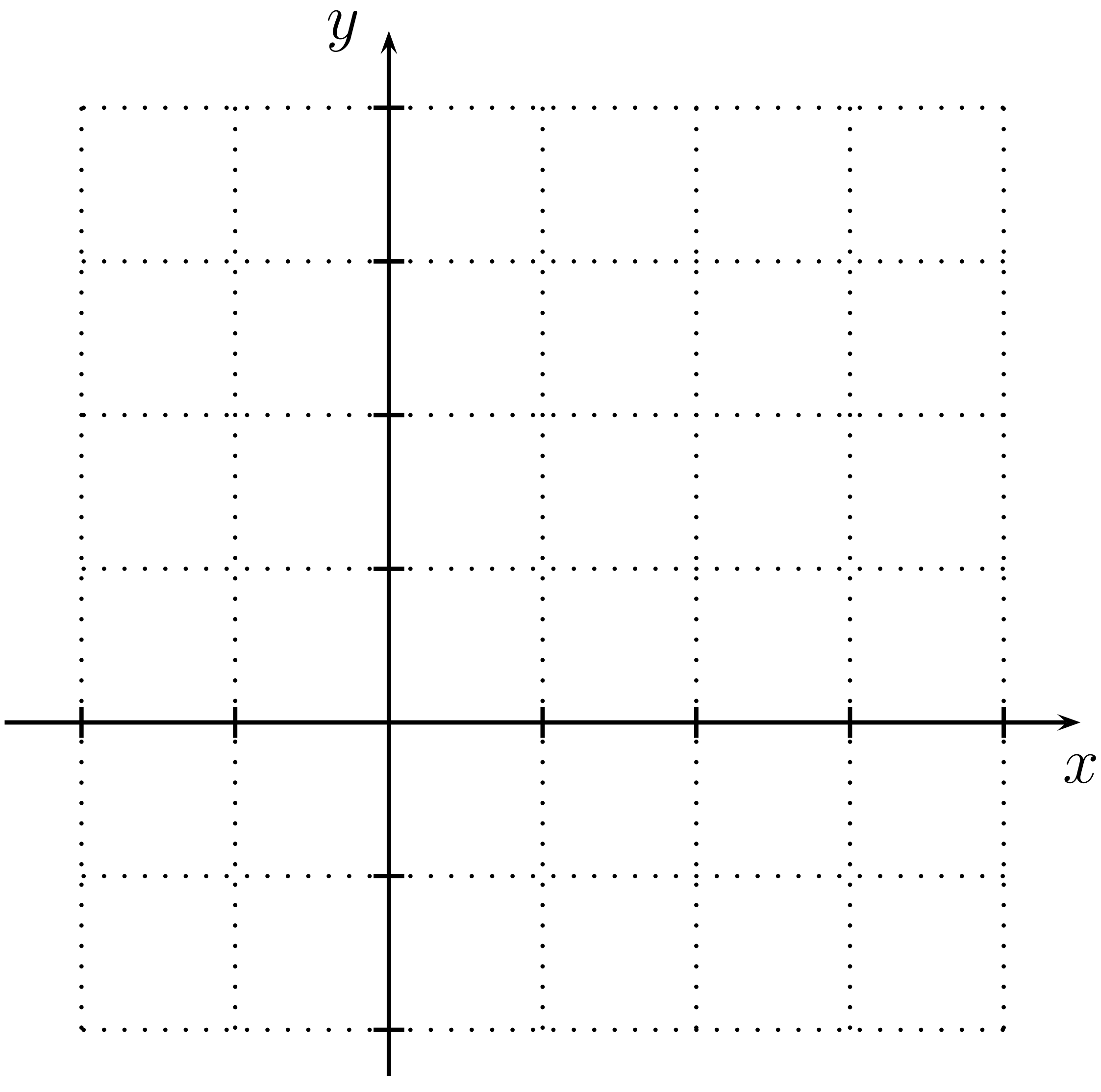
Graph the function f when $B = 4$ and $B = -1$.

Find a value of B such that the function is continuous at $x = 1$.

Example 19:

Let $f(x) = [[x]]$ be the function that associates to any value of x the greatest integer less than or equal to x. Compute the $[[0.5]],\ [[1.99]],\ [[2]],\ [[2.01]],\ [[4.87]],\ [[-1.5]]$. Make a graph of the function $y = [[x]]$. Compute $\lim_{x\to 2^-} [[x]]$ and $\lim_{x\to 2^+} [[x]]$.

Example 19 (continued):



Example 20:

Consider the function $f(x) = \sqrt{x}$. What can you say about the tangent line to the graph of f at the point $x = 0$?

Example 21:

Consider the function $f(x) = |x|$. What can you say about the tangent line to the graph of f at the point $x = 0$?

Example 22:

Let

$$f(x) = \begin{cases} x^2, & \text{if } x \leq 2; \\ mx + b, & \text{if } x > 2. \end{cases}$$

Find the values of m and b that make f differentiable at $x = 2$.