You may find the summation formulas useful:

\[
\sum_{k=1}^{n} k = \frac{n(n + 1)}{2} \quad \text{and} \quad \sum_{k=1}^{n} k^2 = \frac{n(n + 1)(2n + 1)}{6}
\]

1. Write the sum

\[1 + 4 + 9 + 16 + 25 + \cdots + 196 + 225\]

in summation notation. Then use a summation formula to find the value of the sum.

**Solution:** The terms in the sum are the squares of integers, so

\[1 + 4 + 9 + 16 + 25 + \cdots + 196 + 225 = \sum_{k=1}^{15} k^2 = \frac{15 \cdot (15 + 1)(2 \cdot 15 + 1)}{6} = 1240\]

2. Write the sum

\[15 + 20 + 25 + 30 + 35 + 40 + 45 + 50\]

in summation notation. Then use a summation formula to find the value of the sum.

**Solution:**

The terms in the sum are multiples of 5, starting with the third multiple of 5, so

\[15 + 20 + 25 + 30 + 35 + 40 + 45 + 50 = 5 \cdot (3 + 4 + 5 + 6 + 7 + 8 + 9 + 10) = 5 \cdot \sum_{k=3}^{10} k\]

To make use of the summation formulas we require our sums to start at \(k = 1\). But notice our above sum is the same as

\[5 \cdot (3 + 4 + 5 + 6 + 7 + 8 + 9 + 10) = 5 \cdot (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10) - 15\]

\[= 5 \cdot (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10) - 15 = 5 \cdot \frac{10 \cdot 11}{2} - 15 = 260\]

3. (a) Write the sum

\[\sum_{k=4}^{10} (2k + 1)\]

in expanded form.
(b) Write the sum
\[ \sum_{k=1}^{7} (2k + 7) \]
in expanded form.

(c) The above two parts show that a sum can be written in Σ notation in different ways. Find \( b \) so that
\[ \sum_{k=2}^{8} (2k + b) \]
is the same as the sums in parts (a) and (b).

Solution:
(a)
\[ \sum_{k=4}^{10} (2k + 1) \]
\[ = (2 \cdot 4 + 1) + (2 \cdot 5 + 1) + (2 \cdot 6 + 1) + (2 \cdot 7 + 1) + (2 \cdot 8 + 1) + (2 \cdot 9 + 1) + (2 \cdot 10 + 1) \]
\[ = 9 + 11 + 13 + 15 + 17 + 19 + 21 = 105 \]

(b)
\[ \sum_{k=1}^{7} (2k + 7) \]
\[ = (2 \cdot 1 + 7) + (2 \cdot 2 + 7) + (2 \cdot 3 + 7) + (2 \cdot 4 + 7) + (2 \cdot 5 + 7) + (2 \cdot 6 + 7) + (2 \cdot 7 + 7) \]
\[ = 9 + 11 + 13 + 15 + 17 + 19 + 21 = 105 \]

(c)
\[ \sum_{k=2}^{8} (2k + b) \]
\[ = (2 \cdot 2 + b) + (2 \cdot 3 + b) + (2 \cdot 4 + b) + (2 \cdot 5 + b) + (2 \cdot 6 + b) + (2 \cdot 7 + b) + (2 \cdot 8 + b) \]
\[ = (4 + b) + (6 + b) + (8 + b) + (10 + b) + (12 + b) + (14 + b) + (16 + b) \]
We want this sum to match the sum \( 9 + 11 + 13 + 15 + 17 + 19 + 21 = 105 \). We could proceed in one of two ways (possibly more?) First, we could try to find \( b \) so that the individual terms match up, so that 9 and \( 4 + b \) are the same, 11 and \( 6 + b \) are the same, etc. Doing so, we see that \( b = 5 \). Alternatively, we could match the values of the sums. Now,
\[ (4 + b) + (6 + b) + (8 + b) + (10 + b) + (12 + b) + (14 + b) + (16 + b) \]
\[ = (4 + 6 + 8 + 10 + 12 + 14 + 16) + 7b = 70 + 7b \]
Thus, $70 + 7b = 105$ and so again we find $b = 5$.

4. (Challenge) Evaluate the sum

$$\sum_{k=1}^{40}(2k - 3)^2$$

(Hint: You may need to write the expression $(2k - 3)^2$ in a different form in order to make use of the summation formulas)

**Solution:**

Notice that

$$(2k - 3)^2 = 4k^2 - 12k + 9$$

so

$$\sum_{k=1}^{40}(2k - 3)^2 = \sum_{k=1}^{40}(4k^2 - 12k + 9)$$

$$= 4 \left( \sum_{k=1}^{40}k^2 \right) - 12 \left( \sum_{k=1}^{40}k \right) + \left( \sum_{k=1}^{40}9 \right)$$

$$= 4 \cdot \frac{40 \cdot 41 \cdot 81}{6} - 12 \cdot \frac{40 \cdot 41}{2} + 40 \cdot 9 = 79080$$

**Remark:** Sums like the above appear often in probability and statistics when dealing with standard deviation and variance calculations. This probably means nothing to you right now, but may be relevant a semester or two from now.