MA123 — Elem. Calculus Spring 2016
Exam 1 2016-02-11

Name: SOLUTIONS Sec.: _____

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name on this page, and at the top of page three.

**GOOD LUCK!**

3. (a) (b) (c) (d) **(e)**
4. (a) (b) **(c)** (d) (e)
5. (a) **(b)** (c) (d) (e)
6. (a) (b) (c) (d) **(e)**
7. (a) (b) (c) (d) **(e)**
8. (a) (b) **(c)** (d) (e)
9. (a) **(b)** (c) (d) (e)
10. (a) (b) (c) (d) **(e)**
11. (a) (b) (c) **(d)** (e)
12. **(a)** (b) (c) (d) (e)
13. (a) (b) (c) (d) **(e)**
14. (a) (b) (c) **(d)** (e)
15. (a) (b) (c) **(d)** (e)
16. **(a)** (b) (c) (d) (e)
17. (a) (b) (c) **(d)** (e)
18. (a) (b) **(c)** (d) (e)
19. **(a)** (b) (c) (d) (e)
20. (a) **(b)** (c) (d) (e)

For grading use:

| Multiple Choice | Short Answer |
|---|---|
| (number right) (5 points each) | (out of 10 points) |

| Total | |
|---|---|
| | (out of 100 points) |

**Spring 2016 Exam 1 Short Answer Questions**

*Write answers on this page. You must show appropriate legible work to be sure you will get full credit.*

4 pts 1. Evaluate the one-sided limit $\lim_{x\to 0^-} \frac{|x|}{6x}$.

$$\lim_{x\to 0^-} \frac{|x|}{6x} = \lim_{x\to 0^-} \frac{-x}{6x} \quad (*\text{cancel } x\text{'s})$$

↑ notice $x$ will be negative

$$= \lim_{x\to 0^-} \frac{-1}{6}$$

$$= \boxed{\frac{-1}{6}}$$

6 pts 2. Let $f(x)=x^2+3$. Find a value of $x$ such that the **average rate of change** of $f(x)$ from 1 to $x$ equals 12.

$$AROC = \frac{f(b)-f(a)}{b-a} = \frac{f(x)-f(1)}{x-1}$$

$$= \frac{(x^2+3)-(1^2+3)}{x-1}$$

$$= \frac{x^2+3-4}{x-1} = \frac{x^2-1}{x-1} = \frac{(x+1)(x-1)}{(x-1)}$$

$$= x+1$$

Now set $AROC = 12$

$$x+1 = 12$$

$$\boxed{x = 11}$$

Name: SOLUTIONS

**Multiple Choice Questions**

*Show all your work on the page where the question appears.*
*Clearly mark your answer both on the cover page on this exam*
*and in the corresponding questions that follow.*

3. Solve the equation $t = r + \frac{k}{5}w$ for $w$

**Possibilities:**

(a) $w = \dfrac{5r - 5t}{k}$

(b) $w = \dfrac{k}{5t - 5r}$

(c) $w = \dfrac{5t}{r + k}$

(d) $w = \dfrac{t}{r + \frac{k}{5}}$

(e) $w = \dfrac{5t - 5r}{k}$

$t = r + \frac{k}{5}w$

$t - r = \frac{k}{5}w$

$\frac{5}{k}(t - r) = \frac{5}{k}\left(\frac{k}{5}w\right)$

$\frac{5}{k}(t - r) = w$

4. Evaluate $f(4)$ when $f(x)$ is given by the piecewise definition

$$f(x) = \begin{cases} x^2 - 7 & \text{if } x \leq 3 \\ x - 2 & \text{if } 3 < x \leq 4 \\ x^2 - 5x & \text{if } 4 < x \end{cases}$$

$3 < 4 \leq 4$

* we only need be concerned with middle part of function

**Possibilities:**

(a) $-4$

(b) 7

(c) 2

(d) 9

(e) DNE

$f(4) = 4 - 2 = 2$

5. A train travels from city A to city B, then travels from city B to city C. The train leaves city A at 9:00am and arrives at city B at 11:00am. The train leaves city B at 11:00am and arrives at city C at 4:00pm. The average velocity of the train, while traveling from A to B, was 31 miles per hour. What was the average velocity of the train from city B to city C, given that the average velocity of the train while traveling from A to C was 54 miles per hour?

**Possibilities:**

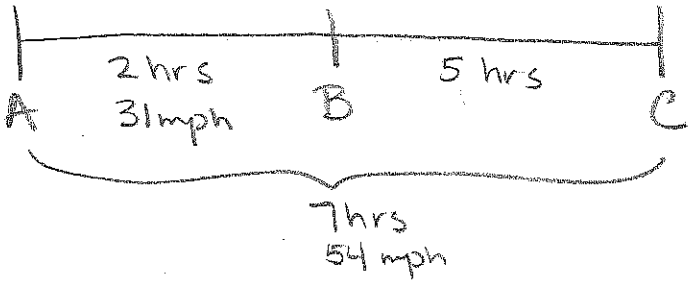
(a) (85/2) miles per hour

(b) (316/5) miles per hour

(c) (321/5) miles per hour

(d) (23/2) miles per hour

(e) 85 miles per hour




from A → B : 2 hrs · 31 mph = 62 miles

from A → C : 7 hrs · 54 mph = 378 miles

thus from B → C : 378 − 62 = 316 miles

So train traveled 316 miles in 5 hrs or $\frac{316}{5}$ mph

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6. If $f(x) = \sqrt{x+3}$ then choose the simplified form of $\frac{f(x+h)-f(x)}{h}$:

**Possibilities:**

(a) $\frac{1}{2}\sqrt{x+h+3} - \frac{1}{2}\sqrt{x+3}$

(b) 1

(c) $\dfrac{\frac{1}{2}}{\sqrt{x+h+3}}$

(d) $\dfrac{h\sqrt{x+3}+\frac{1}{2}}{\sqrt{x+3}}$

(e) $\dfrac{1}{\sqrt{x+h+3}+\sqrt{x+3}}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{x+h+3} - \sqrt{x+3}}{h}$$

$$= \left(\frac{\sqrt{x+h+3} - \sqrt{x+3}}{h}\right) \cdot \left(\frac{\sqrt{x+h+3} + \sqrt{x+3}}{\sqrt{x+h+3} + \sqrt{x+3}}\right)$$

$$= \frac{(x+h+3)-(x+3)}{h\left(\sqrt{x+h+3} + \sqrt{x+3}\right)}$$

$$= \frac{h}{h\left(\sqrt{x+h+3} + \sqrt{x+3}\right)}$$

$$= \frac{1}{\sqrt{x+h+3} + \sqrt{x+3}}$$

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7. Let $f(x) = 3x^2 + 4$. Find a value of $x$ so that $f'(x) = 36$.

**Possibilities:**

(a) $x = 2$

(b) $x = 3$

(c) $x = 4$

(d) $x = 5$

(e) $x = 6$

$f(x) = 3x^2 + 0x + 4$

by rule from Chapter 2,

$f'(x) = 2(3)x + 0 = 6x$

Solve

$f'(x) = 6x = 36$

$x = 6$

8. Let $f(x) = 6x^2 + 4x + 3$. Find a value $c$ between $x = 3$ and $x = 7$, so that the average rate of change of $f(x)$ from $x = 3$ to $x = 7$ is equal to the instantaneous rate of change of $f(x)$ at $x = c$.

**Possibilities:**

(a) 3

(b) 4

(c) 5

(d) 6

(e) 7

$\text{AROC} \Rightarrow \frac{f(7) - f(3)}{7 - 3} = \frac{[6(7)^2 + 4(7) + 3] - [6(3)^2 + 4(3) + 3]}{4}$

$= \frac{325 - 69}{4} = \frac{256}{4} = 64$

$\text{IROC} \Rightarrow f'(x) = 2(6x + 4 = 12x + 4$

let AROC = IROC at $x = c$

$64 = 12c + 4$

$60 = 12c$

$5 = c$

9. If $\lim_{x\to 13} f(x) = 11$ and $\lim_{x\to 13} g(x) = 7$, then what is the value of $\lim_{x\to 13} \dfrac{17f(x)+2}{x+g(x)}$?

**Possibilities:**

(a) 0

(b) $\dfrac{(17)11+2}{13+7}$

(c) the limit is infinity or does not exist

(d) $\dfrac{(17)(11)(13)+2}{13+(7)(13)}$

(e) $\dfrac{11}{7}$

$$\lim_{x\to 13} \frac{17f(x)+2}{x+g(x)}$$

$$= \frac{17 \cdot \lim_{x\to 13} f(x) + 2}{\lim_{x\to 13} x + \lim_{x\to 13} g(x)}$$

$$= \frac{17 \cdot 11 + 2}{13 + 7}$$

10. Find the limit

$$\lim_{x\to 34} \frac{x^2-16}{x-34}$$

plug in 34 $\rightarrow \dfrac{34^2-16}{34-34} = \dfrac{1140}{0}$

"$\frac{a}{0}$" $\rightarrow$ DNE

**Possibilities:**

(a) $\dfrac{16}{34}$

(b) 38

(c) 0

(d) 1

(e) This limit either tends to infinity or this limit fails to exist

11. Find the limit

$$\lim_{x\to 0}\left(\frac{15}{x}+\frac{5x-15}{x}\right)$$

**Possibilities:**

(a) This limit does not exist.

(b) 15

(c) 0

(d) 5

(e) 1

$$= \lim_{x\to 0} \frac{5x}{x} \quad (* \text{ cancel } x\text{'s})$$

$$= \lim_{x\to 0} 5 = 5$$

12. Find the limit

$$\lim_{n\to\infty}\frac{(n+3)^2}{13n^2+11}$$

look at the highest order terms

$$= \lim_{n\to\infty} \frac{n^2}{13n^2} \quad (* \text{ cancel } n^2)$$

$$= \lim_{n\to\infty} \frac{1}{13}$$

$$= \frac{1}{13}$$

**Possibilities:**

(a) $\frac{1}{13}$

(b) The limit does not exist or approaches infinity

(c) $\frac{1}{24}$

(d) $\frac{9}{13}$

(e) $\frac{1}{11}$

13. For the function

$$f(x) = \begin{cases} |3+4x| & \text{if } x < -1 \\ \sqrt{x^2+4} & \text{if } -1 \le x < 2 \\ 9x^2+4x+8 & \text{if } 2 \le x \end{cases}$$

find $\lim_{x \to 3^+} f(x)$

3 ≥ 2 so we're only concerned with bottom function

**Possibilities:**

(a) $\sqrt{13}$

(b) $\sqrt{8}$

(c) 52

(d) 15

(e) 101

Since polynomials are continuous,

$$\lim_{x \to 3^+} f(x) = f(3)$$
$$= 9(3)^2 + 4(3) + 8$$
$$= 101$$

14. The graph of $y = f(x)$ is shown below. Compute $\lim_{x \to 2^-} f(x)$.

approach 2 from the left

**Possibilities:**

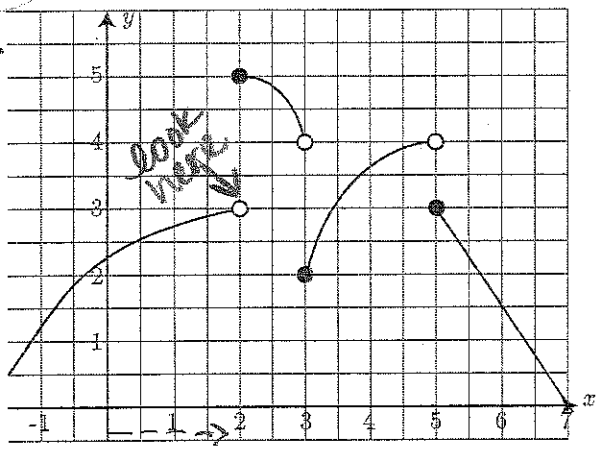
(a) 5

(b) 2

(c) The limit does not exist or approaches infinity

(d) 3

(e) 4



15. Consider the function $f(x) = \begin{cases} x^2 - 3 & \text{if } x < 4 \\ 2x + B & \text{if } x \geq 4 \end{cases}$

Find a value of $B$ so that the function is continuous at $x = 4$.

**Possibilities:**

(a) 2
(b) 3
(c) 4
(d) 5
(e) 6

Continuous at x=4 means function values should be the same on both sides

So at x=4

$x^2 - 3 = 2x + B$

$4^2 - 3 = 2(4) + B$

$13 = 8 + B$

$5 = B$

16. Find the value of $m$ which makes $f(x)$ differentiable everywhere, where

$$f(x) = \begin{cases} x^2, & \text{if } x \leq 3; \\ m(-3 + x) + 9, & \text{if } x > 3 \end{cases}$$

**Possibilities:**

(a) 6
(b) 7
(c) 8
(d) 9
(e) 10

differentiable everywhere means f(x) differentiable at the break point (x=3) derivatives should be the same on both sides

So at x=3

$2x = m$

$2(3) = m$

$6 = m$

17. Find the equation of the tangent line to the graph of the function $f(x) = \frac{1}{x^2+1} + 3$ at $x = 3$. You may use $f'(x) = -\frac{2x}{(x^2+1)^2}$

**Possibilities:**

(a) $y = \frac{31}{10}$

(b) $y = \frac{31}{10}x - \frac{234}{25}$

(c) $y = x^3 + 17$

(d) $y = -\frac{3}{50}x + \frac{82}{25}$

(e) $y = -\frac{3}{50}x + \frac{31}{10}$

* need a point and a slope

point $f(3) = \frac{1}{3^2+1} + 3 = \frac{1}{10} + 3 = \frac{31}{10}$

Slope $f'(3) = \frac{-2(3)}{(3^2+1)^2} = \frac{-6}{100} = \frac{-3}{50}$

tangent line

$y - \frac{31}{10} = \frac{-3}{50}(x-3)$

$y - \frac{31}{10} = \frac{-3}{50}x + \frac{9}{50}$

$y = \frac{-3}{50}x + \frac{164}{50}$

$y = -\frac{3}{50}x + \frac{82}{25}$

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18. Consider the function $f(x) = 6x^2 + 4x + 5$. Its tangent line at $x = 4$ goes through the point $(8, y_1)$ where $y_1$ is:

**Possibilities:**

(a) 117

(b) 100

(c) 325

(d) 52

(e) $-91$

point $f(4) = 6(4)^2 + 4(4) + 5 = 117$ $(4, 117)$

Slope $f'(4)$

$f'(x) = 2(6)x + 4 = 12x + 4$

$f'(4) = 12(4) + 4 = 52$ $m = 52$

tangent line

$y - 117 = 52(x-4)$

$y = 52x - 91$

Now we want the point on the tangent line $(8, y_1)$

$y_1 = 52(8) - 91 = 325$

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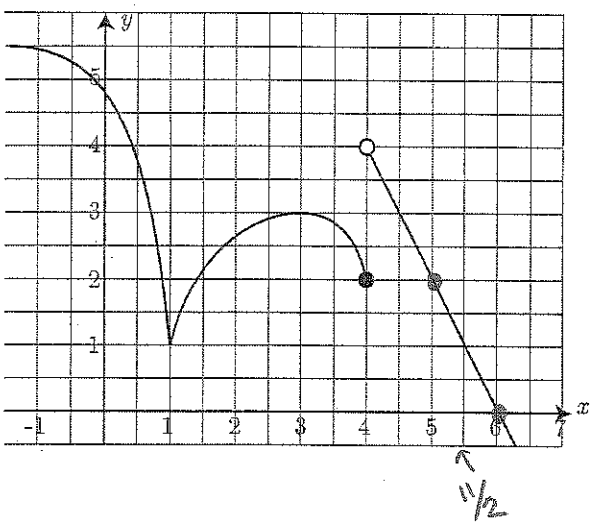
19. The graph of $y = f(x)$ is shown below. $f'(\frac{11}{2})$ is approximately :




**Possibilities:**

(a) $-2$

(b) 2

(c) The limit does not exist or tends to infinity

(d) $-\frac{1}{2}$

(e) $\frac{1}{2}$

$f'(\frac{11}{2})$ is the slope of the tangent line at $x = \frac{11}{2}$

use points (5,2) and (6,0)

$$m = \frac{0-2}{6-5} = \frac{-2}{1} = -2$$

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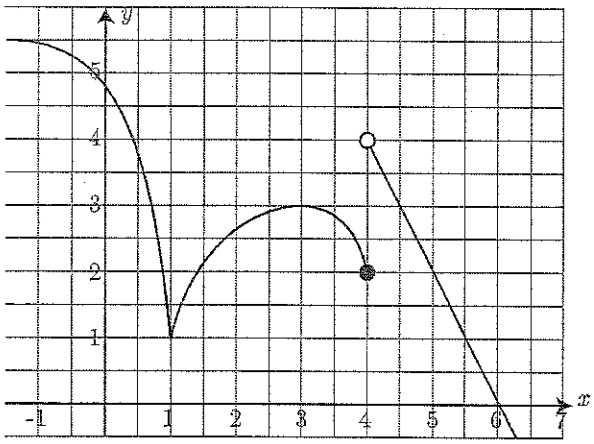
20. The graph of $y = f(x)$ is shown below. The function is differentiable, except at $x =$




**Possibilities:**

(a) x=4 only

(b) x=1 and x=4

(c) x=1, x=3, and x=4

(d) x=1 only

(e) x=1, x=3, x=4, and x=6

look for:

- skips, jumps, holes (not continuous at x=4)
- sharp turns (at x=1)
- vertical tangent lines (none pictured)