MA123 — Elem. Calculus	Spring 2018	Name: Solutions	Sec.:
Exam 1	2018-02-08		

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name and section number on this page.

GOOD LUCK!

3. (a) (b) (c) (d) (e)
4. (a) (b) (c) (d) (e)
5. (a) (b) (c) (d) (e)
6. (a) (b) (c) (d) (e)
7. (a) (b) (c) (d) (e)
8. (a) (b) (c) (d) (e)
9. (a) (b) (c) (d) (e)
10. (a) (b) (c) (d) (e)
11. (a) (b) (c) (d) (e)
12. (a) (b) (c) (d) (e)
13. (a) (b) (c) (d) (e)
14. (a) (b) (c) (d) (e)
15. (a) (b) (c) (d) (e)
16. (a) (b) (c) (d) (e)
17. (a) (b) (c) (d) (e)
18. (a) (b) (c) (d) (e)
19. (a) (b) (c) (d) (e)
20. (a) (b) (c) (d) (e)

For grading use:

Multiple Choice	Short Answer
(number right) (5 points each)	(out of 10 points)

Total	
	(out of 100 points)

Solutions

Spring 2018 Exam 1 Short Answer Questions

Write answers on this page. You must show appropriate legible steps to be sure you will get full credit.

1. Let $f(x)=11x^2+4$. Find a value of x such that the slope of the tangent line to the graph of $f(x)$ equals 77 at that x value. Circle your final answer.

Slope of tangent line $= f'(x) = 22x$ (derivative of ax^2+bx+c is $2ax+b$)

Set $f'(x)=77$ and solve:

$22x = 77$

$x = \frac{77}{22} = \frac{7}{2} = 3.5$

$x = 3.5$

2. Let $f(x)=x^2+3x$. Find the **average rate of change** of $f(x)$ as x changes from x to $x+h$. **Simplify** your answer, and circle your final answer. Show steps clearly.

$\text{AROC} = \frac{f(x+h)-f(x)}{\cancel{x}+h-\cancel{x}}$ (as x changes from x to $x+h$)

$= \frac{(x+h)^2+3(x+h)-(x^2+3x)}{h}$

$= \frac{\cancel{x^2}+2xh+h^2+\cancel{3x}+3h-\cancel{x^2}-\cancel{3x}}{h}$

$= \frac{2xh+h^2+3h}{h} = \frac{\cancel{h}(2x+h+3)}{\cancel{h}}$

$= 2x+h+3$

Name: Solutions

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

3. The expression

$$\sqrt[11]{x^8}$$

is equivalent to which of the following?

Possibilities:

(a) $x^{-\frac{8}{11}}$

(b) $x^{-\frac{11}{8}}$

(c) $x^{\frac{8}{11}}$

(d) x^{88}

(e) $x^{\frac{11}{8}}$

$\sqrt[11]{x^8} = (x^8)^{1/11} = x^{8 \cdot \frac{1}{11}} = x^{\frac{8}{11}}$

4. The graph of $y = f(x)$ is shown below. The expression $f(a) = 0$ is true for which value(s) of a?

Possibilities:

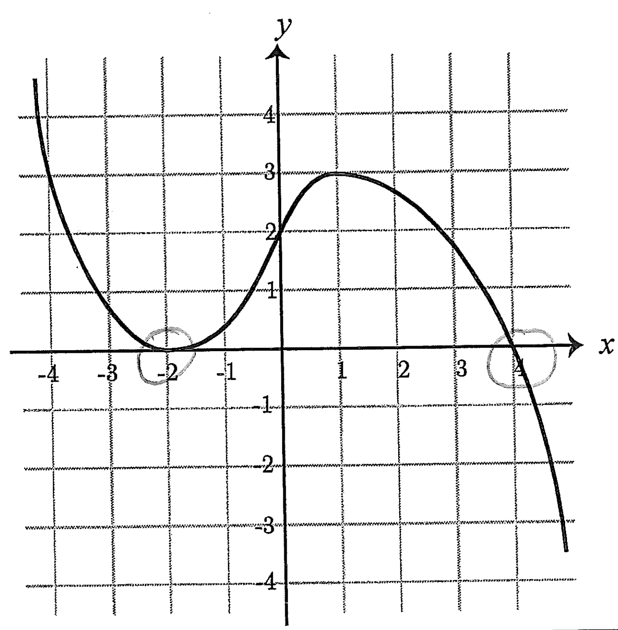
(a) 0

(b) 0, 3

(c) $-2, 1$

(d) $-2, 4$

(e) 2



5. A particle is traveling along a straight line. Its position at time t is given by $s(t) = 9t^2 + 30$. Find the velocity at time $t = 4$.

Possibilities:

(a) 174

(b) 66

(c) 102

(d) 36

(e) 72

velocity is given by $s'(t)$

$s'(t) = 18t + 0 = 18t$

So $s'(4) = 18 \cdot 4 = 72$

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6. If $f(x) = \dfrac{7}{x+2}$ then choose the simplified form of $\frac{f(x+h)-f(x)}{h}$:

Possibilities:

(a) $-\dfrac{7}{(x+h+2)(x+2)}$

(b) $\dfrac{14x+28+7h}{(x+h+2)(x+2)(2x+h)}$

(c) $\dfrac{7}{(x+h+2)(x+2)}$

(d) $-\dfrac{7-h(x+2)^2}{(x+2)^2}$

(e) $-\dfrac{7}{(x+h+2)^2}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{7}{x+h+2} - \frac{7}{x+2}}{h}$$

$$= \frac{\frac{7(x+2)}{(x+h+2)(x+2)} - \frac{7(x+h+2)}{(x+2)(x+h+2)}}{h}$$

$$= \frac{\frac{7x+14}{(x+2)(x+h+2)} - \frac{7x+7h+14}{(x+2)(x+h+2)}}{h}$$

$$= \frac{\cancel{7x}+\cancel{14}-\cancel{7x}-7h-\cancel{14}}{h\cdot(x+2)(x+h+2)} = \frac{-7\cancel{h}}{\cancel{h}(x+2)(x+h+2)} = \frac{-7}{(x+2)(x+h+2)}$$

7. The graph of $y = f(x)$ is shown below. Compute the average rate of change of $f(x)$ from $x = 1$ to $x = 3$.

Possibilities:

(a) $\frac{2}{5}$

(b) 2

(c) $\frac{1}{5}$

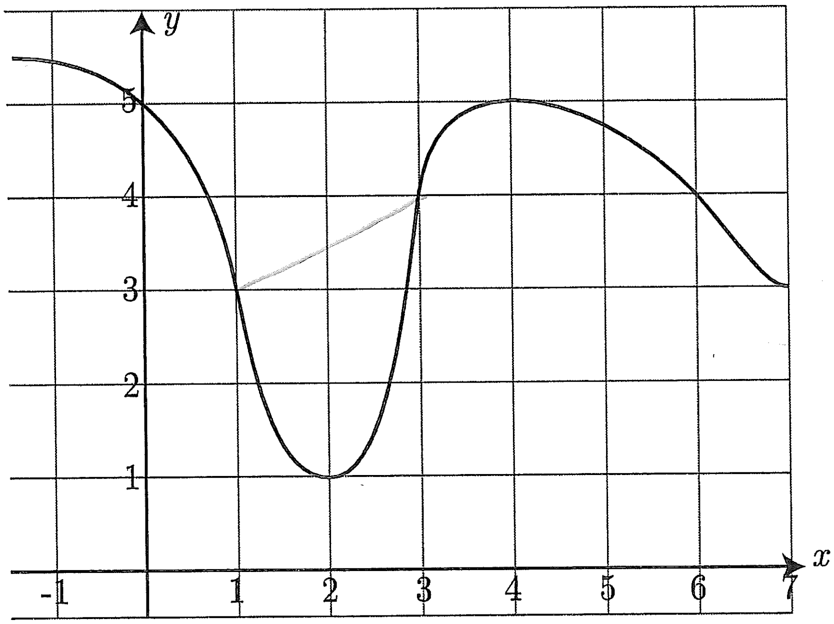
(d) $\frac{1}{2}$

(e) $\frac{2}{3}$

$AROC = \frac{f(3)-f(1)}{3-1}$

$= \frac{4-3}{2}$

$= \frac{1}{2}$



8. Let $f(x) = x^4$. Find a value c between $x = 0$ and $x = 5$, so that the average rate of change of $f(x)$ from $x = 0$ to $x = 5$ is equal to the instantaneous rate of change of $f(x)$ at $x = c$. You may use the fact that $f'(x) = 4x^3$.

Possibilities:

(a) 125

(b) 5/2

(c) $\frac{5}{\sqrt[3]{4}}$

(d) $\frac{5}{\sqrt{3}}$

(e) $\frac{5}{\sqrt{5}}$

$AROC = \frac{f(5)-f(0)}{5-0} = \frac{5^4-0^4}{5} = \frac{5^4}{5} = 5^3 = 125$

Solve AROC from $x=0$ to $x=5$ = IROC of f at $x=c$

$125 = f'(c) = 4c^3$

$\frac{125}{4} = c^3$

$\frac{5}{\sqrt[3]{4}} = c$

9. If $\lim_{x\to 17} f(x) = 5$ and $\lim_{x\to 17} g(x) = 3$, then what is the value of $\lim_{x\to 17} \frac{7f(x)+2}{x+g(x)}$?

Possibilities:

(a) 0

(b) $\frac{5}{3}$

(c) the limit is infinity or does not exist

(d) $\frac{(7)5+2}{17+3}$

(e) $\frac{(7)(5)(17)+2}{17+(3)(17)}$

$$\lim_{x\to 17} \frac{7f(x)+2}{x+g(x)}$$

$$= \frac{\lim_{x\to 17} 7f(x)+2}{\lim_{x\to 17} x+g(x)}$$

$$= \frac{7\cdot 5+2}{17+3}$$

10. Find the limit

$$\lim_{x\to 4} \frac{x^2-2x-8}{x^2-x-12}$$

Possibilities:

(a) 4/7

(b) 5/7

(c) 6/7

(d) 1

(e) This limit does not exist

Plug in $x=4$:

$$\frac{4^2-2\cdot 4-8}{4^2-4-12} = \frac{16-8-8}{16-4-12} = \frac{0}{0}$$

Try factoring/simplifying

$$\lim_{x\to 4} \frac{\cancel{(x-4)}(x+2)}{\cancel{(x-4)}(x+3)} = \lim_{x\to 4} \frac{x+2}{x+3}$$

Plug in $x=4$. $\frac{4+2}{4+3} = \frac{6}{7}$

11. Find the limit

$$\lim_{t\to 0^+} \frac{42t}{\sqrt{t}} = \lim_{t\to 0^+} \frac{42\sqrt{t}\cancel{\sqrt{t}}}{\cancel{\sqrt{t}}}$$

$$= \lim_{t\to 0^+} 42\sqrt{t}$$

Plug in $t=0$: $42\cdot 0 = 0$

Possibilities:

(a) $\frac{21}{\sqrt{t}}$

(b) 0

(c) 42

(d) 21

(e) This limit either tends to infinity or this limit fails to exist

12. Find the limit

$$\lim_{n\to\infty} \frac{(8n+3)^2}{7n^2+5} = \lim_{n\to\infty} \frac{64n^2+48n+9}{7n^2+5}$$

(look at the highest power terms) $= \lim_{n\to\infty} \frac{64n^2}{7n^2} = \lim_{n\to\infty} \frac{64}{7}$

$= \frac{64}{7}$

Possibilities:

(a) $\frac{64}{7}$

(b) $\frac{64}{5}$

(c) $\frac{8}{7}$

(d) The limit does not exist or approaches infinity

(e) $\frac{9}{5}$

13. Given the function $f(x) = \begin{cases} x & \text{if } x \leq 0 \\ 9x+4 & \text{if } x > 0 \end{cases}$ ← Use this one

evaluate the limit as x tends to zero from the right,

$$\lim_{x \to 0+} f(x)$$

$= \lim_{x \to 0^+} 9x+4$

Plug in $x=0$: $9 \cdot 0 + 4 = 4$

Possibilities:

(a) 13

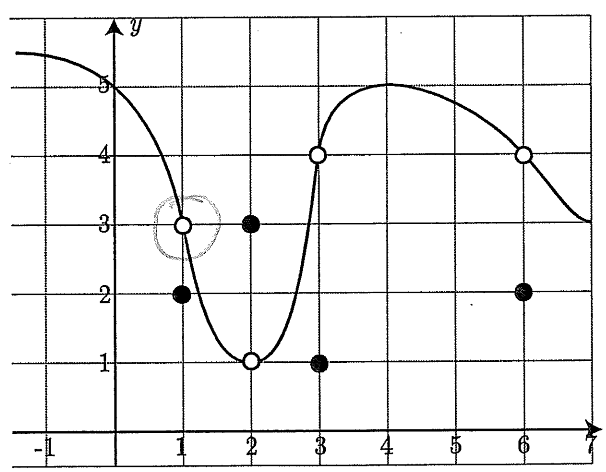
(b) 9

(c) 0

(d) 4

(e) This limit does not exist

14. The graph of $y = f(x)$ is shown below. Compute $\lim_{x \to 1} f(x)$.



Possibilities:

(a) The limit does not exist or approaches infinity

(b) 0

(c) 1

(d) 2

(e) 3

15. Consider the function $f(x) = \begin{cases} x^2 - 7 & \text{if } x < 3 \\ 2x + B & \text{if } x \geq 3 \end{cases}$

Find a value of B so that the function is continuous at $x = 3$.

Possibilities:

(a) -6

(b) -5

(c) -4

(d) -3

(e) -2

Need $\lim_{x\to 3^-} f(x) = \lim_{x \to 3^+} f(x)$.

Plug in $x=3$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} x^2-7 = (3)^2-7 = 9-7=2$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} 2x+B = 2(3)+B = 6+B$.

Plug in $x=3$

Need $2 = 6+B$

$-4 = B$

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16. Determine the value of $f'(-3)$ from the graph of $f(x)$ given here:

Possibilities:

(a) $f'(-3) = 1$

(b) $f'(-3) = -3$

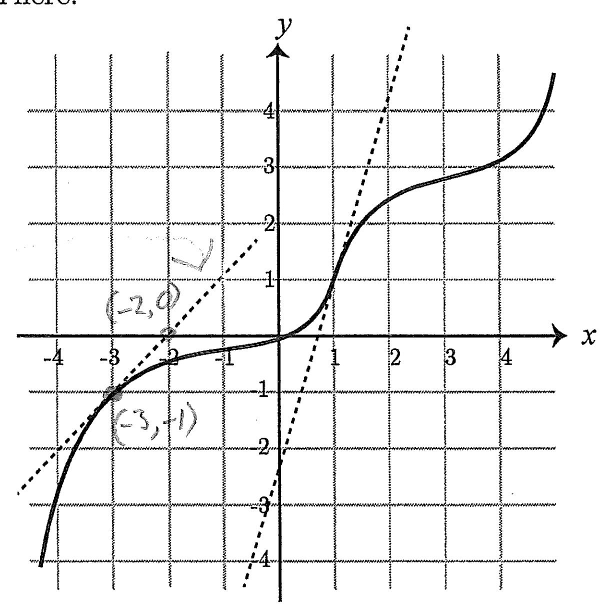
(c) $f'(-3) = -1$

(d) $f'(-3) = 0$

(e) $f'(-3) = 3$

$f'(-3) =$ slope of this line

Slope $= \dfrac{0-(-1)}{-2-(-3)} = \dfrac{1}{-2+3} = \dfrac{1}{1} = 1$



17. For the function $f(x) = (x+10)^2$, find the equation of the tangent line to the graph of f at $x = 3$.

Possibilities:

(a) $y = 6x + 151$

(b) $y = x + 10$

(c) $y = 26x + 91$

(d) $y = 6x + 169$

(e) $y = 26x + 169$

Slope of tangent line $= f'(3)$

$f(x) = x^2 + 20x + 100$

$f'(x) = 2x + 20$

$f'(3) = 2(3) + 20 = 26$

Tangent line given by $y - f(3) = f'(3)(x-3)$

$y - (3+10)^2 = 26(x-3)$

$y - 13^2 = 26x - 78$

$y = 26x - 78 + 169 = 26x + 91$

18. Consider the function $f(x) = x^2 + 7x + 2$. Its tangent line at $x = 3$ goes through the point $(1, y_1)$ where y_1 is:

Possibilities:

(a) 13

(b) 32

(c) 9

(d) 6

(e) -7

First, find equation of tangent line.

Slope is $f'(3)$

$f'(x) = 2x + 7$

$f'(3) = 2(3) + 7 = 13$

$f(3) = 3^2 + 7(3) + 2$

$= 9 + 21 + 2 = 32$

Equation $y - 32 = 13(x-3)$

$y - 32 = 13x - 39$

$y = 13x - 7$

Now plug in $x = 1$ to equation of tangent line.

$y_1 = 13(1) - 7 = 6$

19. The graph of $y = f(x)$ is shown below. The function is **continuous**, except at $x =$

Possibilities:

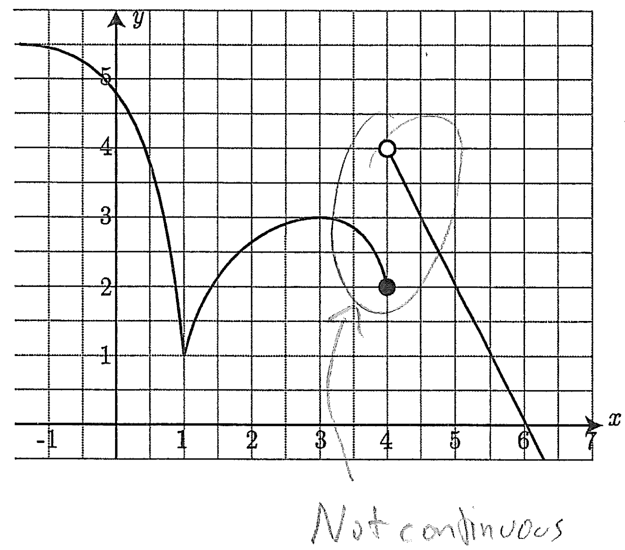
(a) x=1, x=3, and x=4

(b) x=1, x=3, x=4, and x=6

(c) x=1 only

(d) x=1 and x=4

(e) x=4 only



20. The graph of $y = f(x)$ is shown below. The function is **differentiable**, except at $x =$

Possibilities:

(a) x=1 and x=4

(b) x=1, x=3, x=4, and x=6

(c) x=1 only

(d) x=1, x=3, and x=4

(e) x=4 only