MA123 — Elem. Calculus Fall 2018
Exam 2 2018-10-18

Name: SOLUTIONS Sec.: _____

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name and section number on this page.

GOOD LUCK!

3.	a	**b**	c	d	e	12.	**a**	b	c	d	e
4.	a	b	c	**d**	e	13.	a	b	c	**d**	e
5.	**a**	b	c	d	e	14.	**a**	b	c	d	e
6.	a	**b**	c	d	e	15.	a	b	**c**	d	e
7.	a	b	**c**	d	e	16.	a	b	c	d	**e**
8.	a	b	c	d	**e**	17.	**a**	b	c	d	e
9.	a	b	c	**d**	e	18.	a	b	c	**d**	e
10.	a	**b**	c	d	e	19.	a	b	**c**	d	e
11.	a	b	**c**	d	e	20.	a	b	c	d	**e**

For grading use:

Multiple Choice		Short Answer
(number right)	(5 points each)	(out of 10 points)

Total	
	(out of 100 points)

Fall 2018 Exam 2 Short Answer Questions

Write answers on this page. Your work must be clear and legible to be sure you will get full credit.

1. Let $H(x)=(3x+40)\,g(2x)$. Find the derivative, $H'(x)$. **Do not simplify** your answer. Clearly (circle) your final answer.

$H'(x) = (3x+40)\cdot g'(2x)\cdot 2 + g(2x)\cdot 3$

2. The total profit (in dollars) from the sale of x bicycles is given by

$$P(x)=50x-x^2-300$$

Find the **marginal average profit** at a production level of 10 bicycles. **Show all steps clearly** and (circle) your final answer.

Average profit : $\overline{P}(x) = \frac{P(x)}{x} = \frac{50x-x^2-300}{x}$

$= 50 - x - 300x^{-1}$

Marginal average profit : $\overline{P}'(x) = -1 + 300x^{-2}$

Marginal average profit at $x=10$:

$\overline{P}'(10) = -1 + 300(10)^{-2}$

$= -1 + \frac{300}{10^2}$

$= -1 + \frac{300}{100}$

$= -1 + 3$

$= 2 \Rightarrow \$2$

Name: _____

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer on the cover page on this exam.

3. For the function $f(x) = 9x^3 + 7x^2 + 2x + 5$, find the equation of the tangent line to the graph of f at $x = 2$.

Possibilities:

(a) $y = x^3 + 17$

(b) $y = 138x - 167$

(c) $y = 109$

(d) $y = 138x + 109$

(e) $y = 109x - 80$

Slope of tangent line is derivative evaluated at given pt!

$m = f'(2) = 27(2)^2 + 14(2) + 2 = 138$

$y = mx + b$, where $y = f(2)$, $m = 138$, $x = 2$

$f(2) = 9(2)^3 + 7(2)^2 + 2(2) + 5 = 109$

$\Rightarrow 109 = 138(2) + b$

$\Rightarrow 109 - 276 = b$

$-167 = b \Rightarrow$ $y = 138x - 167$

4. Find the derivative, $f'(x)$, if $f(x) = \sqrt[5]{6x^3 + x^2 + 9x + 4}$.

Possibilities:

(a) $(1/5)(6x^3 + x^2 + 9x + 4)^{-1/5}$

(b) $(1/5)(6x^3 + x^2 + 9x + 4)(18x^2 + 2x + 9)$

(c) $(1/5)(18x^2 + 2x + 9)^{-4/5}$

(d) $(1/5)(6x^3 + x^2 + 9x + 4)^{-4/5}(18x^2 + 2x + 9)$

(e) $\sqrt[5]{18x^2 + 2x + 9}$

$f(x) = g(h(x))$

$f'(x) = g'(h(x)) \cdot h'(x)$

$\Rightarrow f(x) = (6x^3 + x^2 + 9x + 4)^{1/5}$

$f'(x) = \frac{1}{5}(6x^3 + x^2 + 9x + 4)^{-4/5} \cdot (18x^2 + 2x + 9)$

5. Find the derivative, $f'(x)$, if $f(x) = \ln(5x + 2) + 50x + 90$.

Possibilities:

(a) $\dfrac{5}{5x + 2} + 50$

(b) $5e^{5x+2} + 50$

(c) $\dfrac{1}{\ln(5x + 2)} \cdot \dfrac{5}{5x + 2} + 50$

(d) $(5x + 2)e^{5x+1} + 50$

(e) $\ln(5x + 2) + 140$

$f'(x) = \frac{1}{5x+2} \cdot 5 + 50$

$= \frac{5}{5x+2} + 50$

6. Suppose $F(x) = g(x)e^{6x}$. If $g(0) = 8$ and $g'(0) = 7$, find $F'(0)$.

Possibilities:

(a) 15

(b) 55

(c) 7

(d) 42

(e) 21

$F(x) = g(x)e^{6x}$

$F'(x) = g'(x)e^{6x} + g(x) \cdot e^{6x} \cdot 6$

$= g'(x)e^{6x} + 6g(x)e^{6x}$

$F'(0) = g'(0)e^{6\cdot 0} + 6g(0)e^{6\cdot 0}$

$= 7 \cdot 1 + 6 \cdot 8 \cdot 1 \quad = 7 + 48 \quad = 55$

---

7. Suppose $g(-3) = -10$ and $g'(-3) = 7$. Find $F'(-3)$ if

$$F(x) = \frac{g(x)}{x^2}$$

Possibilities:

(a) $-\frac{1}{27}$

(b) $-\frac{1}{9}$

(c) $\frac{1}{27}$

(d) $\frac{1}{3}$

(e) $-\frac{7}{3}$

$F'(x) = \dfrac{x^2 \cdot g'(x) - g(x) \cdot 2x}{(x^2)^2}$

$F'(-3) = \dfrac{(-3)^2 \cdot 7 - (-10) \cdot 2(-3)}{((-3)^2)^2}$

$= \dfrac{9 \cdot 7 - (-10 \cdot 6)}{(9)^2}$

$= \dfrac{63 - 60}{81} = \dfrac{3}{81} = \dfrac{1}{27}$

---

8. Suppose $H(x) = f(x^2 + g(x))$. If $g(2) = 10$, $g'(2) = 7$, $f'(11) = 15$, and $f'(14) = 17$, then find $H'(2)$.

Possibilities:

(a) $15(14)(4+17)$

(b) 17

(c) 15

(d) $(17)(11) + (14)(15)$

(e) $17(4+7)$

$H'(x) = f'(x^2 + g(x)) \cdot (2x + g'(x))$

$H'(2) = f'(2^2 + 10) \cdot (2 \cdot 2 + 7)$

$= f'(14) \cdot (4+7)$

$= 17 \cdot (4+7)$

---

9. Suppose $F(x) = f(\ln(x))$. If $f(1) = 7$, $f'(1) = 11$, and $f'(0) = 2$, then find $F'(1)$.

Possibilities:

(a) $\ln(2)$

(b) $\ln(7)/11$

(c) $11/7$

(d) 2

(e) $7/11$

$F'(x) = f'(\ln(x)) \cdot \frac{1}{x}$

$F'(1) = f'(\ln(1)) \cdot \frac{1}{1}$

$= f'(0)$

$= 2$

---

10. For the function $f(x) = \begin{cases} x^2 - 4 & x < 10 \\ x^3 - 5 & 10 \le x < 20 \\ \sqrt{x+9} & 20 \le x \end{cases}$, find the slope of the tangent line to the graph of f at $x = 6$.

Possibilities:

(a) 211

(b) 12

(c) 32

(d) $\frac{1}{30}\sqrt{15}$

(e) 108

For $x = 6$ use $f(x) = x^2 - 4$.

$f'(x) = 2x$

$f'(6) = 2 \cdot 6 = 12$

$m = 12$

---

11. Find the derivative, $f'(x)$, if $f(x) = \ln(\ln(4+9x))$.

Possibilities:

(a) $e^{\frac{9}{4+9x}}$

(b) $\left(\frac{9}{4+9x}\right) e^{\ln(4+9x)}$

(c) $\frac{1}{\ln(4+9x)} \cdot \frac{9}{4+9x}$

(d) $\frac{1}{\frac{9}{4+9x}}$

(e) $\frac{1}{\ln(\ln(4+9x))} \cdot \frac{1}{\ln(4+9x)} \cdot \frac{9}{4+9x}$

$f'(x) = \frac{1}{\ln(4+9x)} \cdot \frac{1}{4+9x} \cdot 9$

$= \frac{1}{\ln(4+9x)} \cdot \frac{9}{4+9x}$

---

12. If $f(x) = 7x^8 + 3x^5 + 9x$ then find the third derivative $f'''(x)$:

Possibilities:

(a) $2352x^5 + 180x^2$

(b) $3584x^8 + 375x^5$

(c) $2352x^5 + 180x^2 + 19x$

(d) $392x^6 + 60x^3$

(e) $\dfrac{56x^7 + 15x^4 + 9}{x^2}$

$f(x) = 7x^8 + 3x^5 + 9x$

$f'(x) = 56x^7 + 15x^4 + 9$

$f''(x) = 392x^6 + 60x^3$

$f'''(x) = 2352x^5 + 180x^2$

13. If $f(x) = e^{14x+38}$ then $f''(x) =$

Possibilities:

(a) $(14x + 38)(14x + 37)e^{14x+36} + 14e^{14x+37}$

(b) $(14x + 38)(14x + 37)e^{14x+36}$

(c) $28^2\,(14)^{28}\,(14x + 38)$

(d) $14^2 e^{14x+38}$

(e) 0

$f(x) = e^{14x+38}$

$f'(x) = e^{14x+38} \cdot 14$

$= 14e^{14x+38}$

$f''(x) = 14e^{14x+38} \cdot 14$

$14^2 e^{14x+38}$

14. Find the derivative, $f'(x)$, of $f(x) = \frac{6}{x^{30}}$

Possibilities:

(a) $-180x^{-31}$

(b) $-30x^{-29}$

(c) $-30x^{-31}$

(d) $180x^{29}$

(e) $6/(30\,x^{29})$

$f(x) = \dfrac{6}{x^{30}}$

$= 6x^{-30}$

$f'(x) = -180x^{-31}$

15. The graph of $f(x)$ passes through the point $(0, 29)$. The slope of f at any point P is 2 times the $y-$coordinate of P. Find the value of $f(1)$.

Possibilities:

(a) $29\ln(2)$

(b) $2\ln(29)$

(c) $29e^2$

(d) $2e^{29}$

(e) 58

Let $y = f(x)$.

"slope is 2 times y-coord"

$\Rightarrow y' = 2y$

This means y satisfies the relationship for exponential growth: $y = ce^{2x}$

Since we have the point $(0, 29)$,

$29 = ce^0 \Rightarrow c = 29.$

Thus $f(x) = 29e^{2x}$,

so $f(1) = \boxed{29e^2}$

16. If \$1000 dollars is invested at 6% interest compounded continuously, what is the value of the investment at the end of 4 years?

Possibilities:

(a) \$110023.18

(b) \$786.63

(c) \$4247.35

(d) \$240.00

(e) \$1271.25

$A = Pe^{rt}$

$= 1000e^{0.06(4)}$

$= 1271.249$

$\Rightarrow \boxed{\$1271.25}$

17. If a tank holds 6000 gallons of water, which drains from the bottom of the tank in 30 minutes, then Torricelli's Law give the volume V of water remaining in the tank after t minutes as

$$V = 6000\left(1 - \frac{t}{30}\right)^2.$$

Find the rate at which water is draining out of the tank after 10 minutes.

Possibilities:

(a) $\frac{800}{3}$ gallons per minute

(b) $\frac{400}{3}$ gallons per minute

(c) 400 gallons per minute

(d) $\frac{8000}{3}$ gallons per minute

(e) 8000 gallons per minute

$V = 6000\left(1 - \frac{t}{30}\right)^2$

$V' = 2 \cdot 6000\left(1 - \frac{t}{30}\right) \cdot -\frac{1}{30}$

$= -\frac{12000}{30}\left(1 - \frac{t}{30}\right)$

$= -400\left(1 - t/30\right)$

$V'(10) = -400\left(1 - 10/30\right)$

$= -400(20/30)$

$= -800/3$

The rate of draining water is $800/3$ gal/min.

18. The total cost (in dollars) of producing x machines is

$$C(x) = 1800 + 30x - .1x^2.$$

Use the **marginal cost** to approximate the cost of producing the 31st machine.

Possibilities:

(a) \$2633.90

(b) \$23.80

(c) \$2610.00

(d) \$24.00

(e) \$23.90

31st machine evaluate $x = 30$

marginal cost is the derivative of the cost

$C'(x) = 30 - .2x$

$C'(30) = 30 - .2(30)$

$= 30 - 6 = 24$

At a production level of 30 units, we expect the 31st unit to cost approximately \$24.

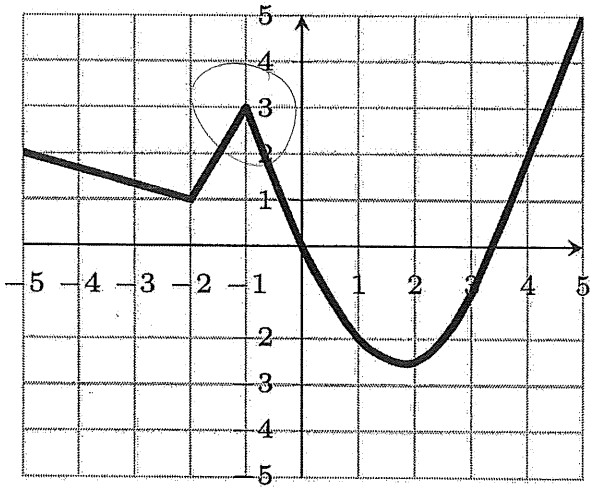
19. The graph of $y = f(x)$ is shown below. The maximum value of $f(x)$ on the interval $[-3, 3]$ occurs at which x?

Possibilities:

(a) 2
(b) -3
(c) -1
(d) -2
(e) 0

Maximum value is the largest value of y for a value of x between -3 and 3.

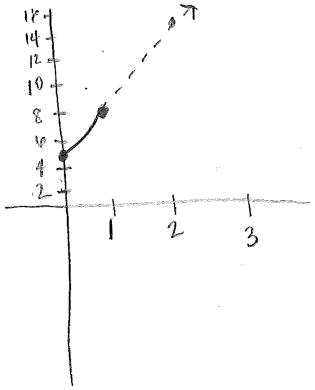
Highest value of y is 3 which occurs at $x = -1$.



20. Find the minimum value of $f(x) = \begin{cases} x^2 + 2x + 5 & \text{if } x \leq 1 \\ 7x + 1 & \text{if } x > 1 \end{cases}$ on the interval [0,10].

Possibilities:

(a) 8
(b) 71
(c) $\frac{7}{2}$
(d) 1
(e) 5



minimum value of 5 occurs at $x = 0$, in the first equation of the piecewise function!

critical number?

$y = x^2 + 2x + 5$

$y' = 2x + 2$

$y' = 0$ when $x = -1$, not in [0,10].

Test endpoints and $x = 1$:

$f(0) = 5$ ← smallest y

$f(1) = 1 + 2 + 5 = 8$

$f(10) = 70 + 1 = 71$