MA123 — Elem. Calculus Spring 2018
Exam 2 2018-3-8

Name: Solutions Sec.: _____

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name and section number on this page.

GOOD LUCK!

3.	(a)	b	c	d	e	12.	a	b	c	d	(e)
4.	a	b	c	(d)	e	13.	a	b	c	(d)	e
5.	a	b	c	d	(e)	14.	(a)	b	c	d	e
6.	a	b	(c)	d	e	15.	a	b	c	(d)	e
7.	(a)	b	c	d	e	16.	a	b	c	d	(e)
8.	a	b	c	d	(e)	17.	(a)	b	c	d	e
9.	a	b	c	(d)	e	18.	a	(b)	c	d	e
10.	(a)	b	c	d	e	19.	a	b	(c)	d	e
11.	a	b	(c)	d	e	20.	a	b	c	(d)	e

For grading use:

Multiple Choice		Short Answer
(number right)	(5 points each)	(out of 10 points)

Total	
	(out of 100 points)

Spring 2018 Exam 2 Short Answer Questions

Write answers on this page. Your work must be clear and legible to be sure you will get full credit.

1. Let $H(x)=\left(x^2+f(x)\right)^3$. Given that $f(1)=-4$ and $f'(1)=6$, find $H'(1)$. Clearly circle your final answer.

Power Rule Chain Rule

$$H'(x) = 3\left(x^2+f(x)\right)^2\left(2x+f'(x)\right)$$

$$H'(1) = 3\left(1^2+f(1)\right)^2\cdot\left(2(1)+f'(1)\right)$$

$$= 3\left(1+-4\right)^2\cdot\left(2+6\right)$$

$$= 3\left(-3\right)^2\cdot 8$$

$$= 3\cdot 9\cdot 8 = 216$$

2. The length of a rectangle is increasing at a rate of 3 cm/min and its width is increasing at a rate of 10 cm/min. When the length is 15 cm and the width is 6cm, how fast is the area of the rectangle increasing? (**Show appropriate calculus steps clearly** and circle your final answer.)

$$A = L\cdot W$$

$$\frac{dA}{dt} = L\frac{dW}{dt} + \frac{dL}{dt}W$$

$$\frac{dA}{dt} = 15\cdot 10 + 3\cdot 6 = 150+18 = 168\ \frac{cm^2}{min}$$

Name: _____

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer on the cover page on this exam.

3. For the function $f(x) = \ln(7x^3 + 2x^2 + 3x + 17)$, find the equation of the tangent line to the graph of f at $x = 0$.

Possibilities:

(a) $y = \frac{3}{17}x + \ln(17)$

(b) $y = \dfrac{21x^3 + 4x^2 + 3x}{7x^3 + 2x^2 + 3x + 17} + \ln(17)$

(c) $y = \ln(17)\, x + 3$

(d) $y = 17$

(e) $y = \frac{17}{3}x + \ln(17)$

slope of tangent line is $f'(0)$

$f'(x) = \dfrac{1}{7x^3+2x^2+3x+17} \cdot (21x^2+4x+3)$

$f'(0) = \frac{3}{17}$

point: $f(0) = \ln(17)$

Equation

$y - f(0) = f'(0) \cdot x$

$y - \ln(17) = \frac{3}{17}x$

$y = \frac{3}{17}x + \ln(17)$

4. Find the derivative, $f'(x)$, if $f(x) = \sqrt[7]{6x^3 + x^2 + 2x + 7}$.

$= (6x^3+x^2+2x+7)^{1/7}$

$f'(x) = \frac{1}{7}(6x^3+x^2+2x+7)^{-6/7} \cdot (18x^2+2x+2)$

Possibilities:

(a) $(1/7)(6x^3 + x^2 + 2x + 7)(18x^2 + 2x + 2)$

(b) $\sqrt[7]{18x^2 + 2x + 2}$

(c) $(1/7)(6x^3 + x^2 + 2x + 7)^{-1/7}$

(d) $(1/7)(6x^3 + x^2 + 2x + 7)^{-6/7}(18x^2 + 2x + 2)$

(e) $\dfrac{\sqrt[7]{18x^2 + 2x + 2}}{\sqrt[7]{6x^3 + x^2 + 2x + 7}}$

5. Find the derivative, $f'(x)$, if $f(x) = e^{8x+3} + 20x + 60$.

$f'(x) = e^{8x+3} \cdot 8 + 20$

Possibilities:

(a) $(8x + 3)e^{8x+2} + 20$

(b) $\dfrac{8}{8x + 3} + 20$

(c) $e^8 + 20$

(d) $\ln(8x + 3) + 80$

(e) $8e^{8x+3} + 20$

6. Suppose $F(x) = g(x) \cdot h(x+2)$. If $g(0) = 9$, $g'(0) = 4$, $h(0) = 3$, $h'(0) = 8$, $h(2) = 5$, and $h'(2) = 6$, find $F'(0)$.

Possibilities:

(a) 60

(b) 84

(c) 74

(d) 35

(e) 128

$F'(x) = g'(x) \cdot h(x+2) + g(x) \cdot h'(x+2) \cdot 1$

$F'(0) = g'(0) \cdot h(2) + g(0) \cdot h'(2)$

$= 4 \cdot 5 + 9 \cdot 6$

$= 20 + 54 = 74$

---

7. Suppose $g(5) = 4$ and $g'(5) = 6$. Find $F'(5)$ if

$$F(x) = \frac{x^3}{g(x)}$$

Possibilities:

(a) $-\frac{225}{8}$

(b) $\frac{3}{2}$

(c) $\frac{225}{8}$

(d) $-\frac{225}{2}$

(e) -18

$F'(x) = \frac{g(x) \cdot 3x^2 - x^3 \cdot g'(x)}{(g(x))^2}$

$F'(5) = \frac{g(5) \cdot 3(5)^2 - (5)^3 \cdot g'(5)}{(g(5))^2} = \frac{4 \cdot 75 - 125 \cdot 6}{42}$

$= \frac{-450}{16} = \frac{-225}{8}$

---

8. Suppose $H(x) = f(x^2 - 15)$. If $f(2) = 9$, $f'(2) = 4$, $f(-11) = 8$, and $f'(-11) = 3$, then find $H'(2)$.

Possibilities:

(a) 3

(b) 36

(c) 16

(d) -44

(e) 12

$H'(x) = f'(x^2 - 15) \cdot 2x$

$H'(2) = f'(2^2 - 15) \cdot 2(2)$

$= f'(-11) \cdot 4 = 3 \cdot 4 = 12$

---

9. Suppose $F(x) = e^{g(x)}$. If $g(9) = 4$ and $g'(9) = 3$, find $F'(9)$.

Possibilities:

(a) $12e^3$

(b) $3e^3$

(c) $4e^3$

(d) $3e^4$

(e) e^4

$F'(x) = g'(x)\, e^{g(x)}$

$F'(9) = g'(9)\, e^{g(9)}$

$= 3e^4$

---

10. For the function $f(x) = \begin{cases} x^2 - 4 & x < 10 \\ x^3 - 7 & 10 \le x < 20 \\ \sqrt{x+9} & 20 \le x \end{cases}$, find the slope of the tangent line to the graph of f at $x = 18$.

Use this one

Possibilities:

(a) 972

(b) 320

(c) $\frac{1}{54}\sqrt{27}$

(d) 5825

(e) 36

$f'(x) = 3x^2$

$f'(18) = 3(18)^2 = 972$

---

11. Find the derivative, $f'(x)$, if $f(x) = \ln(\ln(7+2x))$.

Possibilities:

(a) $\dfrac{1}{\ln(\ln(7+2x))} \cdot \dfrac{1}{\ln(7+2x)} \cdot \dfrac{2}{7+2x}$

(b) $e^{\frac{2}{7+2x}}$

(c) $\dfrac{1}{\ln(7+2x)} \cdot \dfrac{2}{7+2x}$

(d) $\dfrac{1}{\frac{2}{7+2x}}$

(e) $\left(\dfrac{2}{7+2x}\right) e^{\ln(7+2x)}$

$f'(x) = \dfrac{1}{\ln(7+2x)} \cdot \dfrac{1}{7+2x} \cdot 2$

---

12. If $f(x) = 8x^7 + 3x^5 + 2x$ then find the third derivative $f'''(x)$:

Possibilities:

(a) $2744x^7 + 375x^5$

(b) $1680x^4 + 180x^2 + 13x$

(c) $336x^5 + 60x^3$

(d) $\dfrac{56x^6 + 15x^4 + 2}{x^2}$

(e) $1680x^4 + 180x^2$

$f'(x) = 56x^6 + 15x^4 + 2$

$f''(x) = 336x^5 + 60x^3$

$f'''(x) = 1680x^4 + 180x^2$

---

13. If $f(x) = (17x + 31)^{22}$ then $f''(x) =$

Possibilities:

(a) $22^2\,(17)^{22}\,(17x + 31)$

(b) $22(21)17^{20}$

(c) $22\,(17x + 31)^{21}$

(d) $22(21)\,(17x + 31)^{20}\,(17)^2$

(e) 0

$f'(x) = 22(17x+31)^{21} \cdot 17$

$f''(x) = 22 \cdot 21\,(17x+31)^{20} \cdot (17)^2$

---

14. Find the derivative, $f'(x)$, of $f(x) = \frac{9}{x^{40}} = 9 \cdot x^{-40}$

Possibilities:

(a) $-360x^{-41}$

(b) $360x^{39}$

(c) $-40x^{-41}$

(d) $-40x^{-39}$

(e) $9/(40\,x^{39})$

$f'(x) = -40 \cdot 9\,x^{-41} = -360x^{-41}$

---

15. If an amount of x dollars is invested at 5% interest compounded continuously, and at the end of 2 years the value of the investment is \$6000, find x.

Possibilities:

(a) \$4123.61

(b) \$5251.87

(c) \$5316.72

(d) \$5429.02

(e) \$6631.02

$P(t) = P_0 e^{rt}$ $\quad P_0 = x$

$= x e^{.05t}$

$6000 = P(2) = x e^{.05(2)} = x e^{0.1}$

$x \approx \frac{6000}{e^{0.1}} = 5429.02$

16. A bacteria culture starts with 2000 bacteria and doubles after 11 hours. If we express the number of bacteria after t hours as $y(t) = a \cdot e^{kt}$, find the value of k.

Possibilities:

(a) $11/\ln(2)$

(b) $2000/\ln(2)$

(c) $\ln(2)/\ln(11)$

(d) 2000

(e) $\ln(2)/11$

$y(0) = 2000$

$y(11) = 2000 e^{k(11)} = 4000$

$e^{k(11)} = 2$

$11k = \ln(2)$

$k = \frac{\ln(2)}{11}$

17. A sphere is growing so its volume is increasing at a rate of 81 cubic feet per minute. At what rate is the radius changing when its radius is 3 feet?

Possibilities:

(a) $\frac{81}{36\pi}$ feet per minute

(b) $\frac{108\pi}{3}$ feet per minute

(c) $\frac{36\pi}{81}$ feet per minute

(d) $\frac{81}{12\pi}$ feet per minute

(e) 2916π feet per minute

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$$

$$81 = \frac{4}{3}\pi \cdot 3(3)^2 \frac{dr}{dt}$$

$$81 \cdot \frac{3}{4} \cdot \frac{1}{\pi 27} = \frac{dr}{dt}$$

$$\frac{81}{36\pi} \text{ feet per min} = \frac{dr}{dt}$$

---

18. A street light is at the top of a 17 foot tall pole. A child who is 4 feet tall runs away from the pole with a speed of 7ft/sec along a straight path. How fast is the tip of his shadow moving when he is 49 feet from the base of the pole?

Possibilities:

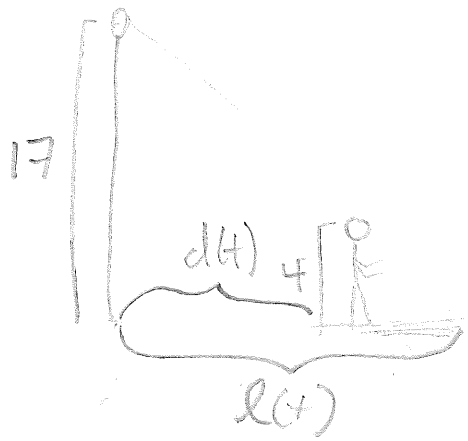
(a) $\frac{119}{49}$ feet per second

(b) $\frac{119}{13}$ feet per second

(c) $\frac{119}{4}$ feet per second

(d) $\frac{28}{17}$ feet per second

(e) $\frac{343}{17}$ feet per second



$$d'(t) = 7$$

$$\frac{\ell(t) - d(t)}{\ell(t)} = \frac{4}{17}$$

$$17\ell(t) - 17d(t) = 4\ell(t)$$

$$13\ell(t) = 17d(t)$$

$$13\ell'(t) = 17d'(t)$$

$$\ell'(t) = \frac{17}{13}(7)$$

$$= \frac{119}{13} \text{ feet per second}$$

---

19. The graph of $y = f(x)$ is shown below. What is the maximum value of $f(x)$ on the interval $[-3, 4]$?

Possibilities:

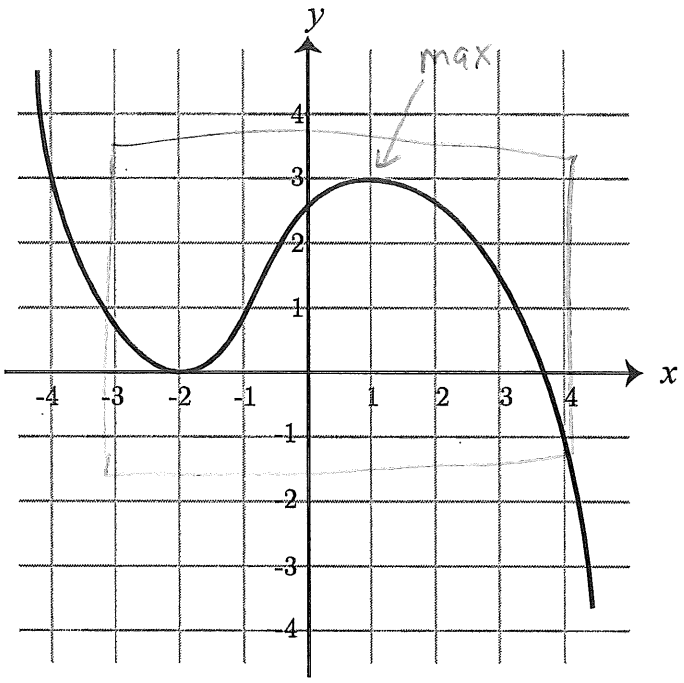
(a) 4

(b) 2

(c) 3

(d) -1

(e) 0



20. Find the minimum value of $g(t) = t^3 - 48t + 50$ on the interval $[-2, 5]$.

Possibilities:

(a) 138

(b) 178

(c) -65

(d) -78

(e) -36

Find critical points, where $g'(t) = 0$

$g'(t) = 3t^2 - 48 = 0$

$3t^2 = 48$

$t^2 = 16$

$t = 4$ (-4 is outside interval)

Check critical point & endpoints

$g(4) = 4^3 - 48(4) + 50 = -78$ ← minimum

$g(-2) = (-2)^3 - 48(-2) + 50 = 138$

$g(5) = 5^3 - 48(5) + 50 = -65$

Some Formulas

1. **Areas:**
   (a) Triangle $A = \frac{bh}{2}$
   (b) Circle $A = \pi r^2$
   (c) Rectangle $A = lw$
   (d) Trapezoid $A = \frac{h_1 + h_2}{2} b$
2. **Volumes:**
   (a) Rectangular Solid $V = lwh$
   (b) Sphere $V = \frac{4}{3}\pi r^3$
   (c) Cylinder $V = \pi r^2 h$
   (d) Cone $V = \frac{1}{3}\pi r^2 h$