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**GOOD LUCK!**

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For grading use:

Multiple Choice	Short Answer
(number right) (5 points each)	(out of 10 points)

Total	
	(out of 100 points)

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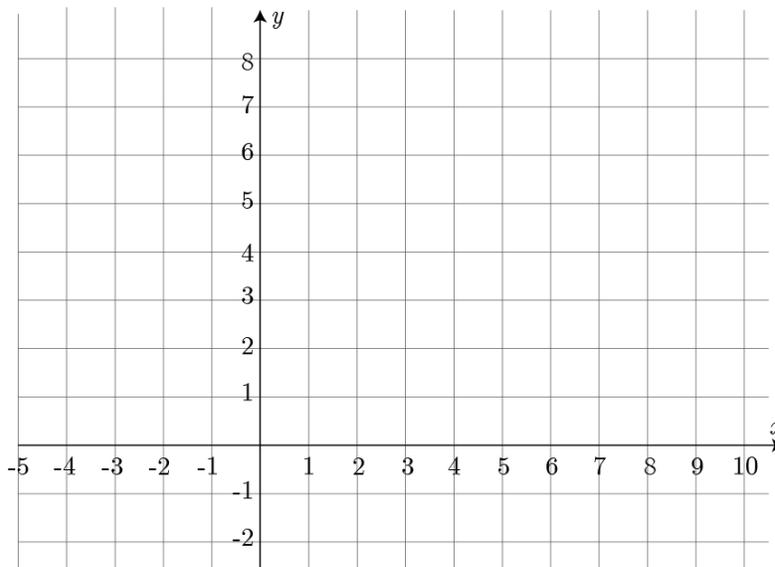
Fall 2015 Exam 3 Short Answer Questions

Write your answers on this page.

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1. Sketch the graph of a **continuous** function  $y = f(x)$  which satisfies the following:

$$f'(x) > 0 \text{ for } x < 3; \quad f'(x) < 0 \text{ for } x > 3; \quad f''(x) < 0 \text{ for } x < 7 \quad f''(x) > 0 \text{ for } x > 7.$$



2. Find the largest possible product you can form from two non-negative numbers whose sum is 53. You must *clearly use calculus* to find and justify your answer.

Largest Possible Product: \_\_\_\_\_

Name: \_\_\_\_\_

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**Multiple Choice Questions**

*Show all your work on the page where the question appears.  
Clearly mark your answer both on the cover page on this exam  
and in the corresponding questions that follow.*

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3. Where is the function  $f(t) = t^3 - 9t^2 - 48t + 1$  decreasing?

**Possibilities:**

- (a)  $f(t)$  is always decreasing
  - (b)  $t < 3$
  - (c)  $t < -2$  and  $t > 8$
  - (d)  $-2 < t < 8$
  - (e)  $t > 3$
- 

4. Where is the function  $f(t) = t^3 - 9t^2 - 48t + 1$  concave up?

**Possibilities:**

- (a)  $f(t)$  is always concave up
  - (b)  $t < 3$
  - (c)  $-2 < t < 8$
  - (d)  $t > 3$
  - (e)  $t < -2$  and  $t > 8$
-

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5. If  $g'(t) = 4 - t^2$ , where is the function  $g(t)$  decreasing?

**Possibilities:**

- (a)  $t > 0$
- (b)  $t < 0$
- (c)  $-2 < t < 2$
- (d)  $f(t)$  is always decreasing
- (e)  $t < -2$  and  $t > 2$

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6. If  $g'(t) = 4 - t^2$ , where is the function  $g(t)$  concave up?

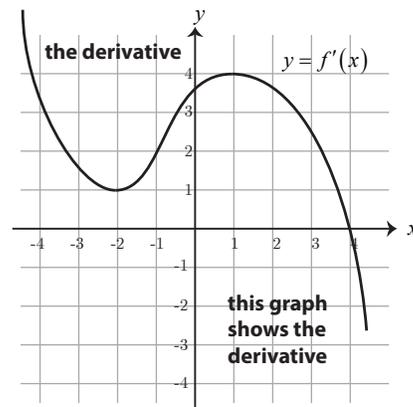
**Possibilities:**

- (a)  $t < 0$
- (b)  $t > 0$
- (c)  $t < -2$  and  $t > 2$
- (d)  $f(t)$  is always concave up
- (e)  $-2 < t < 2$

- 
7. The following is the graph of the derivative,  $f'(x)$ , of the function  $f(x)$ .  
Where is the regular function  $f(x)$  decreasing?

**Possibilities:**

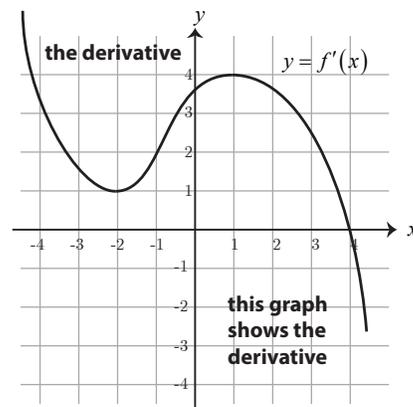
- (a)  $(-\infty, -1)$
- (b)  $(-2, 1)$
- (c)  $(4, \infty)$
- (d)  $(-\infty, -2)$  and  $(1, \infty)$
- (e)  $(-\infty, 4)$



- 
8. The following is the graph of the derivative,  $f'(x)$ , of the function  $f(x)$ .  
Where is the regular function  $f(x)$  concave up?

**Possibilities:**

- (a)  $(-\infty, -1)$
- (b)  $(-2, 1)$
- (c)  $(4, \infty)$
- (d)  $(-\infty, -2)$  and  $(1, \infty)$
- (e)  $(-\infty, 4)$



- 
9. An open box is to be made out of 8-inch by 18-inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. If we find the dimensions of the resulting box that has the largest volume, what is its height?

**Possibilities:**

- (a) 1.33 inches
- (b) 1.43 inches
- (c) 1.53 inches
- (d) 1.63 inches
- (e) 1.73 inches

- 
10. A car rental agency rents 190 cars per day at a rate of \$29 dollars per day. For each 1 dollar increase in the daily rate, 3 fewer cars are rented. At what rate should the cars be rented to produce maximum income?

**Possibilities:**

- (a) \$46.57 per day
- (b) \$46.37 per day
- (c) \$46.17 per day
- (d) \$46.77 per day
- (e) \$46.97 per day

- 
11. Suppose the derivative of  $H(s)$  is given by  $H'(s) = -(s^2 + 4)(s^2 + 2)$ . Find the value of  $s$  in the interval  $[-10, 10]$  where  $H(s)$  takes on its minimum.

**Possibilities:**

- (a)  $-2$
- (b)  $4$
- (c)  $-10$
- (d)  $2$
- (e)  $10$

- 
12. Suppose you estimate the area under the graph of  $f(x) = x^3$  from  $x = 5$  to  $x = 25$  by adding the areas of the rectangles as follows: partition the interval into 20 equal subintervals and use the right endpoint of each interval to determine the height of the rectangle. What is the area of the 7th rectangle?

**Possibilities:**

- (a)  $25$
- (b)  $105400$
- (c)  $1728$
- (d)  $\frac{6095}{4}$
- (e)  $1331$

- 
13. Estimate the area under the graph of  $-x^2 + 20x$  for  $x$  between 4 and 10, by using a partition that consists of 3 equal subintervals of  $[4, 10]$  and use the right endpoint of each subinterval as a sample point.

**Possibilities:**

- (a) 528
- (b) 560
- (c) 688
- (d) 280
- (e) 488

- 
14. A train travels in a straight westward direction along a track. The speed of the train varies, but it is measured at regular time intervals of  $1/10$  hour. The measurements for the first half hour are:

time	0	.1	.2	.3	.4	.5
speed	0	4	10	14	20	23

Estimate the total distance (in miles) traveled by the train during the first half hour by assuming the speed is a linear function of  $t$  on the subintervals. The speed in the table is given in miles per hour. Use all six speed measurements in your estimate.

**Possibilities:**

- (a) 7.00 miles
- (b) 5.95 miles
- (c) 7.10 miles
- (d) 11.50 miles
- (e) 2.00 miles

- 
15. One way to approximate  $\int_A^{59} e^{19-2x} dx$  is with the sum  $\sum_{k=1}^{200} ((\Delta x) \cdot (e^{19-2(9+k\Delta x)}))$  where  $\Delta x = \frac{1}{4}$ .  
What is the best value of  $A$  to use?

**Possibilities:**

- (a) 1.359140914
- (b)  $\frac{1}{4}$
- (c) 200
- (d) 0.01
- (e) 9

- 
16. Suppose you estimate the integral

$$\int_3^8 x^2 dx$$

by adding the areas of  $n$  rectangles of equal length, and using the right endpoint of each subinterval to determine the height of each rectangle. If the sum you evaluate is written as

$$\sum_{k=1}^n \frac{A}{n} \left( 3 + k \frac{A}{n} \right)^2$$

What value should be used for  $A$ ?

**Possibilities:**

- (a) 5
- (b) 3
- (c) 8
- (d) 11
- (e)  $\frac{485}{3}$

---

17. Evaluate the difference of sums

$$\left( \sum_{k=1}^{40000} (6k^3 + 5) \right) - \left( \sum_{k=3}^{40000} (6k^3 + 5) \right)$$

**Possibilities:**

- (a) 800020000
- (b) 3840000000000005
- (c) 0
- (d) 64
- (e)  $\infty$

---

18. Evaluate the sum

$$\sum_{k=1}^N (11k^2)$$

**Possibilities:**

- (a)  $11N^2$
- (b)  $11N^2 - 11$
- (c)  $11 \frac{N(N+1)(2N+1)}{6}$
- (d)  $11N^2 + 11$
- (e)  $11 \frac{N(N+1)}{2}$

---

19. Evaluate the sum  $5 + 10 + 15 + 20 + 25 + 30 + 35 + 40 + 45 + 50 + \cdots + 370 + 375$ .

**Possibilities:**

- (a) 717250
- (b) 70500
- (c) 1020
- (d) 140625
- (e) 14250

---

20. Evaluate the sum  $\frac{1}{13} + \frac{4}{13} + \frac{9}{13} + \frac{16}{13} + \frac{25}{13} + \frac{36}{13} + \frac{49}{13} + \frac{64}{13} + \frac{81}{13} + \frac{100}{13} + \cdots + \frac{841}{13} + \frac{900}{13}$ .

**Possibilities:**

- (a)  $\frac{9455}{13}$
  - (b)  $\frac{13515}{13}$
  - (c)  $\frac{2126}{13}$
  - (d)  $\frac{810000}{169}$
  - (e)  $\frac{410850}{169}$
-

## Some Formulas

### 1. Summation formulas:

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

### 2. Areas:

(a) Triangle  $A = \frac{bh}{2}$

(b) Circle  $A = \pi r^2$

(c) Rectangle  $A = lw$

(d) Trapezoid  $A = \frac{h_1 + h_2}{2} b$

### 3. Volumes:

(a) Rectangular Solid  $V = lwh$

(b) Sphere  $V = \frac{4}{3}\pi r^3$

(c) Cylinder  $V = \pi r^2 h$

(d) Cone  $V = \frac{1}{3}\pi r^2 h$

### 4. Distance:

(a) Distance between  $(x_1, y_1)$  and  $(x_2, y_2)$

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

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