MA123 — Elem. Calculus	Fall 2015	Name: SOLUTIONS	Sec.: ______
Exam 3	2015-11-19		

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name on this page, and at the top of page three.

GOOD LUCK!

3. (a) (b) (c) **(d)** (e)
4. (a) (b) (c) **(d)** (e)
5. (a) (b) (c) (d) **(e)**
6. **(a)** (b) (c) (d) (e)
7. (a) (b) **(c)** (d) (e)
8. (a) **(b)** (c) (d) (e)
9. (a) (b) (c) (d) **(e)**
10. (a) (b) **(c)** (d) (e)
11. (a) (b) (c) (d) **(e)**
12. (a) (b) **(c)** (d) (e)
13. (a) **(b)** (c) (d) (e)
14. (a) **(b)** (c) (d) (e)
15. (a) (b) (c) (d) **(e)**
16. **(a)** (b) (c) (d) (e)
17. (a) (b) (c) **(d)** (e)
18. (a) (b) **(c)** (d) (e)
19. (a) (b) (c) (d) **(e)**
20. **(a)** (b) (c) (d) (e)

For grading use:

Multiple Choice		Short Answer
(number right)	(5 points each)	(out of 10 points)

Total	
	(out of 100 points)

Fall 2015 Exam 3 Short Answer Questions

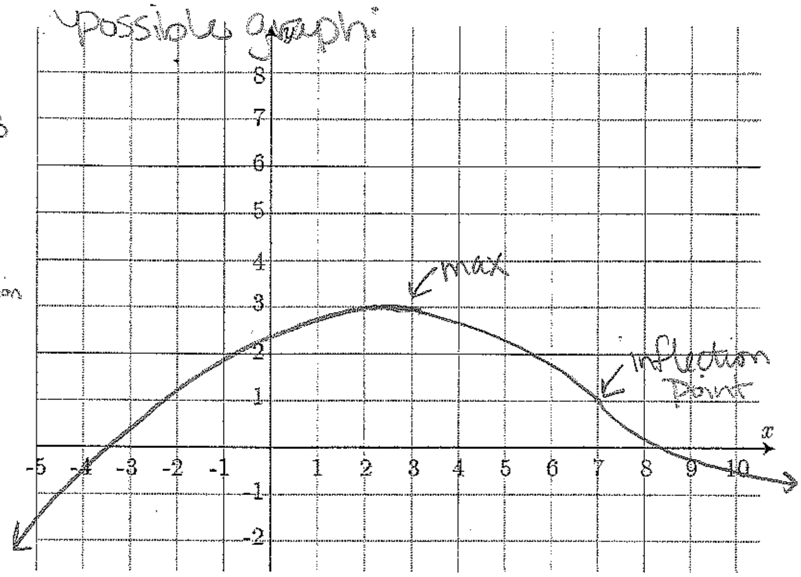
Write your answers on this page.

1. Sketch the graph of a **continuous** function $y = f(x)$ which satisfies the following:

   $f'(x) > 0$ for $x < 3$; $f'(x) < 0$ for $x > 3$; $f''(x) < 0$ for $x < 7$ $f''(x) > 0$ for $x > 7$.

$\left.\begin{array}{l} f(x) \text{ increasing for } x<3 \\ f(x) \text{ decreasing for } x>3 \end{array}\right\}$ max at $x=3$

$\left.\begin{array}{l} f(x) \text{ concave down for } x<7 \\ f(x) \text{ concave up for } x>7 \end{array}\right\}$ inflection pt at $x=7$



2. Find the largest possible product you can form from two non-negative numbers whose sum is 53. *You must* **clearly use calculus** *to find and justify your answer.*

let $x, y \geq 0$

$x + y = 53 \longrightarrow y = 53 - x$

then want to find max $x \cdot y = x(53-x)$

if $f(x) = x(53-x) = 53x - x^2$

then max occurs when $f'(x) = 0$ and $x \in [0, 53]$

$f'(x) = 53 - 2x$

$53 - 2x = 0$

$-2x = -53$

$x = 53/2$

Check: $f(0) = 0(53) = 0$

$f(\frac{53}{2}) = \frac{53}{2}(53 - \frac{53}{2}) = \frac{53}{2} \cdot \frac{53}{2} = 702.25 \leftarrow$ max

$f(53) = 53(0) = 0$

Largest Possible Product: 702.25

Name: SOLUTIONS

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

3. Where is the function $f(t) = t^3 - 9t^2 - 48t + 1$ decreasing?

Possibilities:

(a) $f(t)$ is always decreasing
(b) $t < 3$
(c) $t < -2$ and $t > 8$
(d) $-2 < t < 8$
(e) $t > 3$

$f(t)$ decreases when $f'(t) < 0$

$f'(t) = 3t^2 - 18t - 48$

$3t^2 - 18t - 48 = 0$

$3(t^2 - 6t - 16) = 0$

$3(t-8)(t+2) = 0$

$t = 8, -2$

$f'(t)$: + | −2 | − | 8 | +

check −3, check 0, check 9

decreasing on $(-2, 8)$

4. Where is the function $f(t) = t^3 - 9t^2 - 48t + 1$ concave up?

Possibilities:

(a) $f(t)$ is always concave up
(b) $t < 3$
(c) $-2 < t < 8$
(d) $t > 3$
(e) $t < -2$ and $t > 8$

$f(t)$ concave up when $f''(t) > 0$

from above, we know $f'(t) = 3t^2 - 18t - 48$

$f''(t) = 6t - 18$

$6t - 18 = 0$

$6t = 18$

$t = 3$

$f''(t)$: − | 3 | +

check 0, check 4

concave up $(3, \infty)$

5. If $g'(t) = 4 - t^2$, where is the function $g(t)$ decreasing?

Possibilities:

(a) $t > 0$

(b) $t < 0$

(c) $-2 < t < 2$

(d) $f(t)$ is always decreasing

(e) $t < -2$ and $t > 2$

$g(t)$ decreasing when $g'(t) < 0$

$4 - t^2 = 0$

$4 = t^2$

$\pm 2 = t$

$g'(t)$: $-$ | -2 | $+$ | 2 | $-$

check -3 — check 0 — check 3

$g(t)$ decreasing on $(-\infty, -2) \cup (2, \infty)$

6. If $g'(t) = 4 - t^2$, where is the function $g(t)$ concave up?

Possibilities:

(a) $t < 0$

(b) $t > 0$

(c) $t < -2$ and $t > 2$

(d) $f(t)$ is always concave up

(e) $-2 < t < 2$

$g(t)$ concave up when $g''(t) > 0$

$g''(t) = -2t$

$-2t = 0$

$t = 0$

$g''(t)$: $+$ | 0 | $-$

check -1 — check 1

$g(t)$ concave up on $(-\infty, 0)$

7. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$.
Where is the regular function $f(x)$ decreasing?

Possibilities:

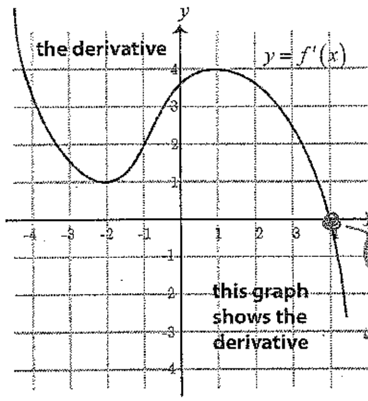
(a) $(-\infty, -1)$

(b) $(-2, 1)$

(c) $(4, \infty)$

(d) $(-\infty, -2)$ and $(1, \infty)$

(e) $(-\infty, 4)$



$f'(x) < 0$

$f(x)$ decreasing when $f'(x) < 0$

so I'm looking for x values where $f'(x) < 0$

$x \in (4, \infty)$

8. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$.
Where is the regular function $f(x)$ concave up?

Possibilities:

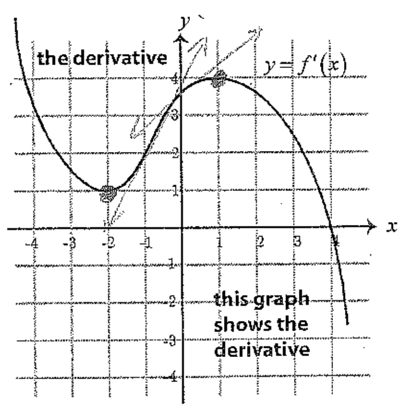
(a) $(-\infty, -1)$

(b) $(-2, 1)$

(c) $(4, \infty)$

(d) $(-\infty, -2)$ and $(1, \infty)$

(e) $(-\infty, 4)$



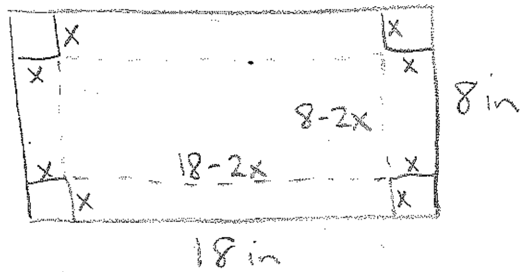
$f(x)$ concave up when $f''(x) > 0$

So I'm looking for where the slope of the tangent line (the derivative) of $f'(x)$ is positive

$x \in (-2, 1)$

9. An open box is to be made out of 8-inch by 18-inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. If we find the dimensions of the resulting box that has the largest volume, what is its height?

$x > 0$
$18 - 2x > 0$
$x < 9$
$8 - 2x > 0$
$x < 4$
thus $x \in [0, 4]$



Possibilities:

(a) 1.33 inches
(b) 1.43 inches
(c) 1.53 inches
(d) 1.63 inches
(e) 1.73 inches

$V = l \cdot w \cdot h$
$= (18-2x)(8-2x)(x) = (144 - 52x + 4x^2)(x)$
$= 144x - 52x^2 + 4x^3$

$V' = 144 - 104x + 12x^2 = 4(36 - 26x + 3x^2)$

$V' = 0$ when $x = \dfrac{26 \pm \sqrt{26^2 - 4(3)(36)}}{2(3)} = \dfrac{26 \pm \sqrt{244}}{6} \approx \cancel{6.934}$ not in domain or 1.729 inches

10. A car rental agency rents 190 cars per day at a rate of \$29 dollars per day. For each 1 dollar increase in the daily rate, 3 fewer cars are rented. At what rate should the cars be rented to produce maximum income?

I = income
n = # cars
P = price/car
then $I = n \cdot p$

Possibilities:

(a) \$46.57 per day
(b) \$46.37 per day
(c) \$46.17 per day
(d) \$46.77 per day
(e) \$46.97 per day

let x = # \$1 increases then

$I = (190 - 3x)(29 + x)$
$= 5510 + 103x - 3x^2$

then $I'(x) = 103 - 6x$
$103 - 6x = 0$
$103 = 6x$
$17.166 = \dfrac{103}{6} = x$

thus cars should be rented at \$29 + 17.17
= \$46.17 per day

11. Suppose the derivative of $H(s)$ is given by $H'(s) = -(s^2+4)(s^2+2)$. Find the value of s in the interval $[-10, 10]$ where $H(s)$ takes on its minimum.

Possibilities:

(a) -2

(b) 4

(c) -10

(d) 2

(e) 10

$H'(s) = \underbrace{-}_{\text{negative}}\underbrace{(s^2+4)}_{\text{positive}}\underbrace{(s^2+2)}_{\text{positive}}$

thus $H'(s) < 0$ for all s.

so $H(s)$ is always decreasing

So its minimum must occur at the right most point of the interval.

$(s=10)$

12. Suppose you estimate the area under the graph of $f(x) = x^3$ from $x = 5$ to $x = 25$ by adding the areas of the rectangles as follows: partition the interval into 20 equal subintervals and use the right endpoint of each interval to determine the height of the rectangle. What is the area of the 7th rectangle?

Possibilities:

(a) 25

(b) 105400

(c) 1728

(d) $\frac{6095}{4}$

(e) 1331

Δx = width of subintervals $= \frac{25-5}{20} = \frac{20}{20} = 1$

x_7 = rt. of 7^{th} subinterval $= 5 + k\Delta x$

$= 5 + 7(1) = 12$

then Area$(\square) = f(x_7)\cdot\Delta x = f(12)\cdot 1 = f(12)$

$= (12)^3 = 1728$

13. Estimate the area under the graph of $-x^2 + 20x$ $= f(x)$ for x between 4 and 10, by using a partition that consists of 3 equal subintervals of $[4, 10]$ and use the right endpoint of each subinterval as a sample point.

$\Delta x = \frac{10-4}{3} = \frac{6}{3} = 2$

$X_k = 4 + k(2)$ so $X_1 = 6,\ X_2 = 8,\ X_3 = 10$

$$\text{Area} = f(x_1)\cdot 2 + f(x_2)\cdot 2 + f(x_3)\cdot 2$$
$$= f(6)\cdot 2 + f(8)\cdot 2 + f(10)\cdot 2$$
$$= (-6^2+20(6))\cdot 2 + (-8^2+20(8))\cdot 2 + (-10^2+20(10))\cdot 2$$
$$= 84\cdot 2 + 96\cdot 2 + 100\cdot 2 = 168 + 192 + 200$$
$$= 560$$

Possibilities:

(a) 528
(b) 560
(c) 688
(d) 280
(e) 488

14. A train travels in a straight westward direction along a track. The speed of the train varies, but it is measured at regular time intervals of 1/10 hour. The measurements for the first half hour are:

time	0	.1	.2	.3	.4	.5
speed	0	4	10	14	20	23

Estimate the total distance (in miles) traveled by the train during the first half hour by assuming the speed is a linear function of t on the subintervals. The speed in the table is given in miles per hour. Use all six speed measurements in your estimate.

Possibilities:

(a) 7.00 miles
(b) 5.95 miles
(c) 7.10 miles
(d) 11.50 miles
(e) 2.00 miles

Will estimate using the average speed over each interval.

$$\text{distance} = \frac{(0+4)}{2}(.1) + \frac{(4+10)}{2}(.1) + \frac{(10+14)}{2}(.1) + \frac{(14+20)}{2}(.1) + \frac{(20+23)}{2}(.1)$$
$$= .2 + .7 + 1.2 + 1.7 + 2.15$$
$$= 5.95 \text{ miles}$$

15. One way to approximate $\int_A^{59} e^{19-2x}\, dx$ is with the sum $\sum_{k=1}^{200} \left((\Delta x) \cdot \left(e^{19-2(9+k\Delta x)}\right)\right)$ where $\Delta x = \frac{1}{4}$. What is the best value of A to use?

$\Delta x = \frac{1}{4} = \frac{59-A}{200}$ So $(59-A)(4) = 200$

$59 - A = 50$

$A = 9$

Possibilities:

(a) 1.359140914

(b) $\frac{1}{4}$

(c) 200

(d) 0.01

(e) 9

16. Suppose you estimate the integral

$$\int_3^8 x^2\, dx$$

by adding the areas of n rectangles of equal length, and using the right endpoint of each subinterval to determine the height of each rectangle. If the sum you evalute is written as

$$\sum_{k=1}^{n} \frac{A}{n}\left(3 + k\frac{A}{n}\right)^2$$

What value should be used for A?

$\Delta x = \frac{8-3}{n} = \frac{5}{n} = \frac{A}{n}$

So $A = 5$

Possibilities:

(a) 5

(b) 3

(c) 8

(d) 11

(e) $\frac{485}{3}$

17. Evaluate the difference of sums

$$\left(\sum_{k=1}^{40000}(6k^3+5)\right)-\left(\sum_{k=3}^{40000}(6k^3+5)\right)$$

$$=\sum_{k=1}^{2}(6k^3+5)$$

$$=\left[6(1)^3+5\right]+\left[6(2)^3+5\right]$$

$$=11+53$$

$$=64$$

Possibilities:

(a) 800020000

(b) 384000000000005

(c) 0

(d) 64

(e) ∞

18. Evaluate the sum

$$\sum_{k=1}^{N}(11k^2)$$

$$=11\sum_{k=1}^{N}k^2$$

$$=11\left(\frac{N(N+1)(2N+1)}{6}\right)$$

Possibilities:

(a) $11N^2$

(b) $11N^2-11$

(c) $11\dfrac{N(N+1)(2N+1)}{6}$

(d) $11N^2+11$

(e) $11\dfrac{N(N+1)}{2}$

19. Evaluate the sum $\underbrace{5 + 10 + 15 + 20 + 25 + 30 + 35 + 40 + 45 + 50 + \cdots + 370 + 375}$.

Possibilities:

(a) 717250

(b) 70500

(c) 1020

(d) 140625

(e) 14250

5k for k=1 to k=75

$$\sum_{k=1}^{75} 5k = 5\sum_{k=1}^{75} k = 5\left(\frac{75(76)}{2}\right)$$

$$= 14,250$$

20. Evaluate the sum $\frac{1}{13} + \frac{4}{13} + \frac{9}{13} + \frac{16}{13} + \frac{25}{13} + \frac{36}{13} + \frac{49}{13} + \frac{64}{13} + \frac{81}{13} + \frac{100}{13} + \cdots + \frac{841}{13} + \frac{900}{13}$. $= \frac{30^2}{13}$

$1 \leq k \leq 30$

Possibilities:

(a) $\frac{9455}{13}$

(b) $\frac{13515}{13}$

(c) $\frac{2126}{13}$

(d) $\frac{810000}{169}$

(e) $\frac{410850}{169}$

$$= \sum_{k=1}^{30} \frac{k^2}{13}$$

$$= \frac{1}{13}\sum_{k=1}^{30} k^2 = \frac{1}{13}\left(\frac{30(31)(61)}{6}\right) = \frac{9455}{13}$$

Some Formulas

1. **Summation formulas:**

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

2. **Areas:**

   (a) Triangle $A = \frac{bh}{2}$

   (b) Circle $A = \pi r^2$

   (c) Rectangle $A = lw$

   (d) Trapezoid $A = \frac{h_1 + h_2}{2} b$

3. **Volumes:**

   (a) Rectangular Solid $V = lwh$

   (b) Sphere $V = \frac{4}{3}\pi r^3$

   (c) Cylinder $V = \pi r^2 h$

   (d) Cone $V = \frac{1}{3}\pi r^2 h$

4. **Distance:**

   (a) Distance between (x_1, y_1) and (x_2, y_2)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$