MA123 — Elem. Calculus Fall 2017
Exam 3 2017-11-16

Name: Solutions **Sec.:** ________

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

[✓ a] b c d e

You have two hours to do this exam. Please write your name and section number on this page.

GOOD LUCK!

3. a b c d [✓ e]
4. [✓ a] b c d e
5. a b c [✓ d] e
6. a [✓ b] c d e
7. a b [✓ c] d e
8. a b c d [✓ e]
9. a [✓ b] c d e
10. a b c [✓ d] e
11. a b c d [✓ e]
12. a b [✓ c] d e
13. [✓ a] b c d e
14. a b c d [✓ e]
15. a [✓ b] c d e
16. a b [✓ c] d e
17. a b c [✓ d] e
18. [✓ a] b c d e
19. a b [✓ c] d e
20. a b c d [✓ e]

For grading use:

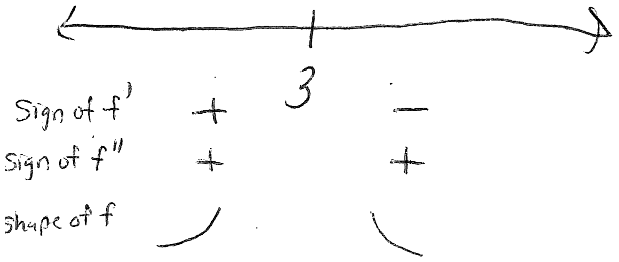
Multiple Choice	Short Answer
(number right) (5 points each)	(out of 10 points)

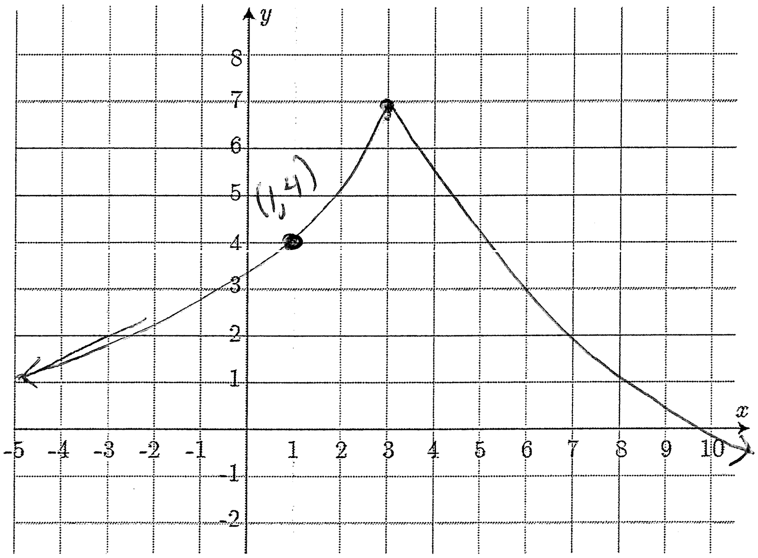
Total	
	(out of 100 points)

Fall 2017 Exam 3 Short Answer Questions

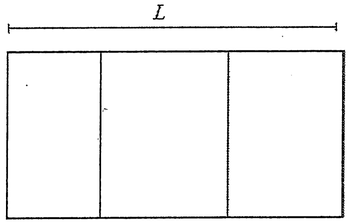
Write answers on this page. Your work must be clear and legible to be sure you will get full credit.

1. Sketch the graph of a **continuous** function $y = f(x)$ which satisfies $f(1) = 4$, $f'(x) > 0$ on $(-\infty, 3)$, $f'(x) < 0$ on $(3, \infty)$, and $f(x)$ is concave up on both $(-\infty, 3)$ and $(3, \infty)$.





2. A farmer builds a rectangular grid of pens with 1 row and 3 columns using 950 feet of fencing. Find the dimensions (overall length and width) that will maximize the total area of the pen. *You must **clearly use calculus** to find **and justify** your answer.*



$4w + 2L = 950$

$2L = 950 - 4w$

$L = 475 - 2w$

$A = Lw$

Plug in $L = 475 - 2w$:

$A = (475 - 2w)w$

$A = 475w - 2w^2$

$A' = 475 - 4w$

A' is never undefined

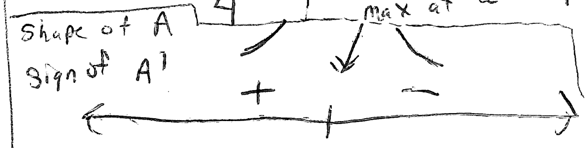
Set $A' = 0$

$0 = 475 - 4w$

$4w = 475$

$w = \frac{475}{4}$

Test



since A changes from increasing to decreasing

$w = 1$ $\frac{475}{4}$ $w = 150$

$A'(1) = 475 - 4$
$= 471$ ~~[illegible]~~
positive

$A'(150) = 475 - 4(150)$
$= 475 - 600$
$= -125$
negative

$L = 475 - 2\left(\frac{475}{4}\right) = \frac{950}{2} - \frac{475}{2} = \frac{475}{2}$

Width W: $\frac{475}{4}$ ft   Overall Length L: $\frac{475}{2}$ ft

Name: Solutions

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

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3. Where is the function $f(t) = (t - 69)^{-2}$ decreasing?

Possibilities:

(a) $f(t)$ is always decreasing except at t = 69

(b) $f(t)$ is never decreasing

(c) $t < 69$

(d) $-1 < t < 69$

[✓ (e) $t > 69$]

$f'(t) = -2(t-69)^{-3}(1)$ ← Chain Rule

$$f'(t) = \frac{-2}{(t-69)^3}$$

~~Set~~ This is undefined when $t = 69$.

Set $f'(t) = 0$

$$(t-69)^3 \cdot \frac{-2}{(t-69)^3} = 0 \cdot (t-69)^3$$

$$\underbrace{-2 = 0}_{\text{no solution}}$$

f	incr.	decr.
sign of f'	+	−
	$t=68$	69	$t=70$

$f'(68) = \frac{-2}{(-1)^3} = \frac{-2}{-1} = 2$

$f'(70) = \frac{-2}{(1)^3} = -2$

f is decreasing on $(69, \infty)$ or $\boxed{t > 69}$

---

4. Where is the function $f(t) = t^4 - 16t^3 - 3$ concave up?

Possibilities:

[✓ (a) $t < 0$ and $t > 8$]

(b) $0 < t < 8$

(c) $t < 12$

(d) $t > 12$

(e) $f(t)$ is always concave up

$f'(t) = 4t^3 - 48t^2$

$f''(t) = 12t^2 - 96t$

~~Set f''(t)=0~~

~~$12t^2 - 96t = 0$~~

Find where $f''(t)$ is undefined or equal to 0

$f''(t)$ is never undefined.

Set $f''(t) = 0$

$12t^2 - 96t = 0$

$12t(t-8) = 0$

$t = 0, 8$

f	conc. up	conc. down	conc. up
sign of f''	+	−	+
	$t=-1$	$t=1$	$t=9$
	0	8	

$f''(-1) = 12(-1)(-9) = 108$

$f''(1) = 12(1)(-7) = -84$

$f''(9) = 12(9)(1) = 108$

f is concave up when $\boxed{t<0 \text{ or } t>8}$

---

5. Suppose the derivative of $g(t)$ is $g'(t) = 15(t-5)(t-9)^2$. Find the interval(s) of values of t in which g is decreasing.

when $g'(t) < 0$

Possibilities:

- (a) $(9, \infty)$
- (b) $(5, 9)$
- (c) $(5, 9) \cup (15, \infty)$
- [✓ (d) $(-\infty, 5)$]
- (e) $(-\infty, 5) \cup (9, \infty)$

Set $g'(t) = 0$

$0 = 15(t-5)(t-9)^2$

$t = 5, 9$

g' is never undefined

	$t=4$	5	$t=6$	9	$t=10$
Sign of g'	$-$		$+$		$+$
g	decr.		incr.		incr.

$g'(4) = 15(-1)(-5)^2$
$= (+)(-)(+)$
$= -$

$g'(6) = 15(1)(-3)^2$
$= (+)(+)(+)$
$= +$

$g'(10) = 15(5)(1)^3$
$= (+)(+)(+)$
$= +$

$\boxed{g \text{ is decreasing on } (-\infty, 5)}$

---

6. Suppose the derivative of $g(t)$ is $g'(t) = 15(t-5)(t-9)$. For t in which interval(s) is g concave up?

Possibilities:

- (a) $(-\infty, 7)$
- [✓ (b) $(7, \infty)$]
- (c) $(5, 7) \cup (9, 15)$
- (d) $(5, 9)$
- (e) $(-\infty, 5) \cup (9, \infty)$

$g'(t) = 15(t^2 - 14t + 45)$

$g''(t) = 15(2t - 14)$

g'' is never undefined

Set $g''(t) = 0$

$15(2t - 14) = 0$

$30(t - 7) = 0$

$t = 7$

	$t=6$	7	$t=9$
Sign of g''	$-$		$+$
g	conc. down		conc. up

$g''(6) = 15(2(6) - 14)$
$= 15(-2) = -30$

$g''(9) = 15(2(9) - 14)$
$= 15(4) = 60$

$\Rightarrow \boxed{g \text{ is conc. up on } (7, \infty)}$

---

7. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$. Where is the regular function $f(x)$ decreasing?

f is decreasing when f' is negative.

Possibilities:

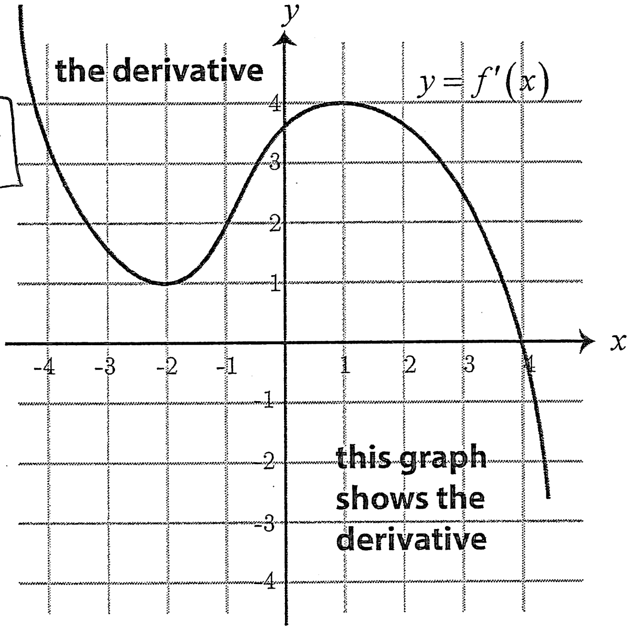
(a) $(-\infty, -1)$

(b) $(-\infty, 4)$ This happens on $\boxed{(4,\infty)}$

[✓ (c) $(4, \infty)$]

(d) $(-2, 1)$

(e) $(-\infty, -2)$ and $(1, \infty)$



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8. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$. Where is the regular function $f(x)$ concave up?

f is concave up when f' is increasing.

Possibilities:

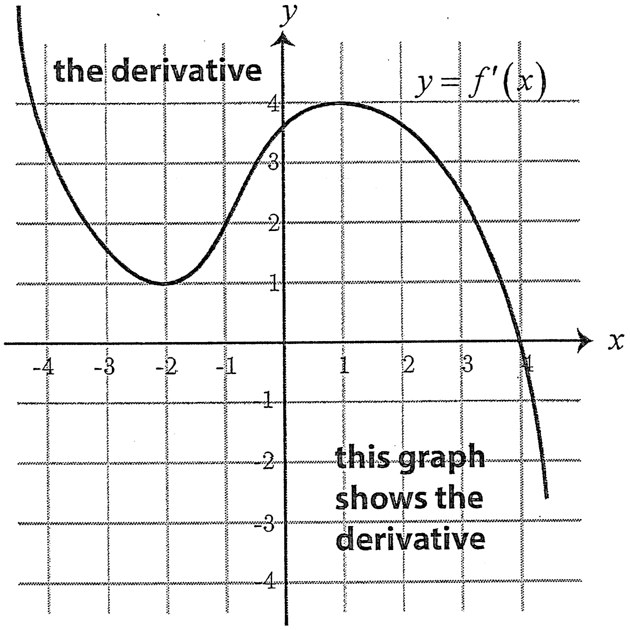
(a) $(-\infty, -1)$

(b) $(-\infty, -2)$ and $(1, \infty)$ This happens on $\boxed{(-2,1)}$

(c) $(-\infty, 4)$

(d) $(4, \infty)$

[✓ (e) $(-2, 1)$]



9. Find the critical numbers of the function $f(x) = 7xe^{19x}$.

Possibilities:

(a) $-\frac{7}{19}, 0$

[✓ (b) $-\frac{1}{19}$]

(c) $-\frac{7}{19}$

(d) $-\frac{1}{19}, 0, e^{19}$

(e) 0

Product Rule and Chain Rule

$f'(x) = 7x\left(e^{19x}\right)' + e^{19x}(7x)'$

$f'(x) = 7xe^{19x}(19) + e^{19x}(7)$

f' is never undefined

Set $f'(x) = 0$

$0 = 19(7)xe^{19x} + 7e^{19x}$

$0 = 7e^{19x}(19x+1)$

e^{19x} is always positive

$\Rightarrow 19x + 1 = 0$

$\Rightarrow 19x = -1$

$\Rightarrow \boxed{x = -\frac{1}{19}}$

---

10. Consider the graph of the original function, $f(x)$.
For this function, what are the signs of $f'(-3)$ and $f''(-3)$?

Possibilities:

(a) $f'(-3) = 0$ and $f''(-3) < 0$

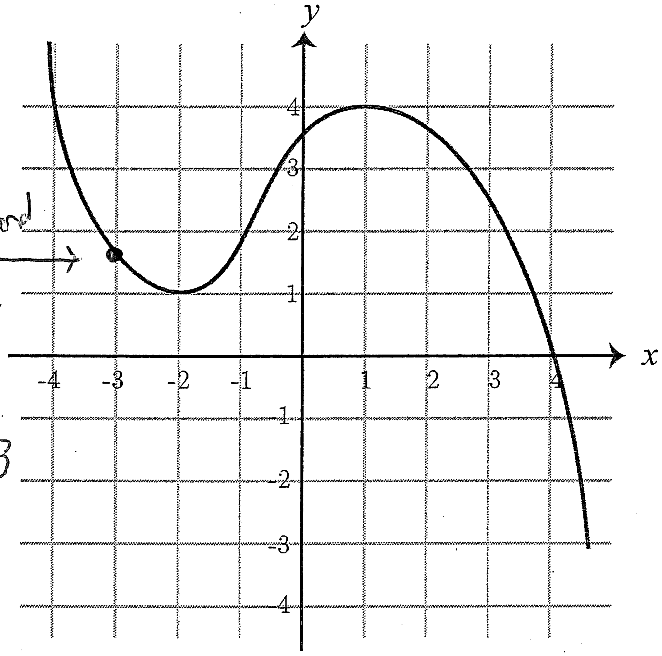
(b) $f'(-3) > 0$ and $f''(-3) < 0$

(c) $f'(-3) < 0$ and $f''(-3) < 0$

[✓ (d) $f'(-3) < 0$ and $f''(-3) > 0$]

(e) $f'(-3) > 0$ and $f''(-3) > 0$

decreasing and concave up at this point →



$f'(-3) < 0$ since f is decr. at $x = -3$

$f''(-3) > 0$ since f is conc. up at $x = -3$

11. The manager of a large apartment complex knows from experience that 110 units will be occupied if the rent is 460 dollars per month. A market survey suggests that, on the average, one additional unit will remain vacant for each 7 dollar increase in rent. Similarly, one additional unit will be occupied for each 7 dollar decrease in rent. What rent should the manager charge to maximize revenue?

Possibilities:

(a) $614.20 per month
(b) $614.60 per month
(c) $615.80 per month
(d) $615.60 per month
[✓ (e) $615.00 per month]

$R = NP$

R = revenue
N = # of units occupied
P = rent

Find an equation involving P and N (linear equation)

Point $(N,P) = (110,460)$

Notice if $N = 111$, then $P = \$453 \Rightarrow (111,453)$

$\text{Slope} = \frac{-7}{1} = -7$

equation: $P - 460 = -7(N-110) = -7N + 770$

$P = -7N + 1230$

$R = N(-7N+1230) = -7N^2 + 1230N$

$R' = -14N + 1230$ ← this is never undefined

$0 = -14N + 1230$

$14N = 1230 \Rightarrow N = \frac{1230}{14}$

≈ 87.86

Test

	$N=1$	$\frac{1230}{14}$	$N=100$
sign of R'	+		−
R	incr.		decr.

$R'(1) = -14 + 1230$
$= 1216$

$R'(100) = -1400 + 1230$
$= -170$

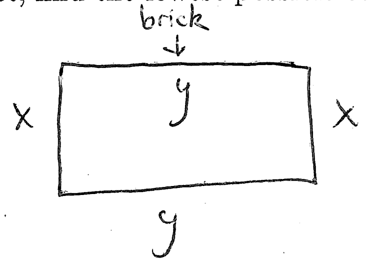
$\Rightarrow R$ has a max at $N = \frac{1230}{14}$

$P = -7\left(\frac{1230}{14}\right) + 1230 = \boxed{\$615}$

12. A landscape architect wishes to enclose a rectangular garden on one side by a brick wall costing $80 per foot, and on the other three sides by a metal fence costing $40 per foot. If the area of the garden is 300 square feet, find the lowest possible cost to enclose the garden.

Possibilities:

(a) $3393.61
(b) $3395.11
[✓ (c) $3394.11]
(d) $3394.61
(e) $3395.61



$C = 80y + 40x + 40x + 40y$

$C = 120y + 80x$

$A = xy = 300$

$y = \frac{300}{x}$

$C = 120\left(\frac{300}{x}\right) + 80x$

$C = 36000x^{-1} + 80x$

$C' = -36000x^{-2} + 80$

$C' = -\frac{36000}{x^2} + 80$

C' is undefined when $x = 0$, but this doesn't make sense for a length to be 0

Set $C' = 0$

$0 = -\frac{36000}{x^2} + 80$

$\frac{36000}{x^2} = 80$

$\Rightarrow 450 = x^2$

$\Rightarrow x = \sqrt{450}$

≈ 21.21

Test

	$x=1$	$\sqrt{450}$	$x=30$
sign of C'	−		+
C	decr.		incr.

$C'(1) = -36000 + 80$
$=$ negative

$C'(30) = -\frac{36000}{900} + 80$
$= -40 + 80$
$= 40$

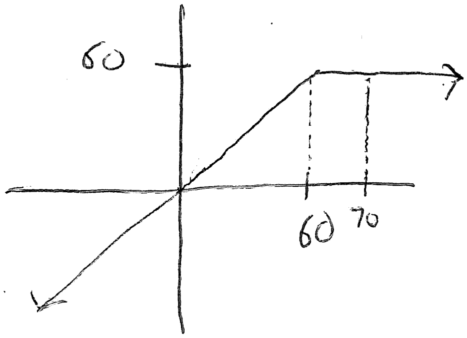
$\Rightarrow C$ has a min. at $x = \sqrt{450}$.

$\Rightarrow C = \frac{36000}{\sqrt{450}} + 80(\sqrt{450}) \approx \boxed{\$3394.11}$

13. Given the function $f(x) = \begin{cases} x & \text{if } x < 60 \\ 60 & \text{if } x \geq 60 \end{cases}$

evaluate the definite integral

$$\int_0^{70} f(x)\,\mathrm{d}x$$



= Signed area under curve

$= \frac{1}{2}(60)(60) + (10)(60)$

$= 1800 + 600$

$= \boxed{2400}$

Possibilities:

[✓ (a) 2400]
(b) 2401
(c) 2402
(d) 2403
(e) 2404

---

14. The graph of $y = f(x)$ shown below consists of straight lines. Evaluate the definite integral $\int_{-3}^{3} f(x)\,\mathrm{d}x$.

$= \underbrace{A_3}_{\text{above axis}} - \underbrace{A_1 - A_2}_{\text{below axis}}$

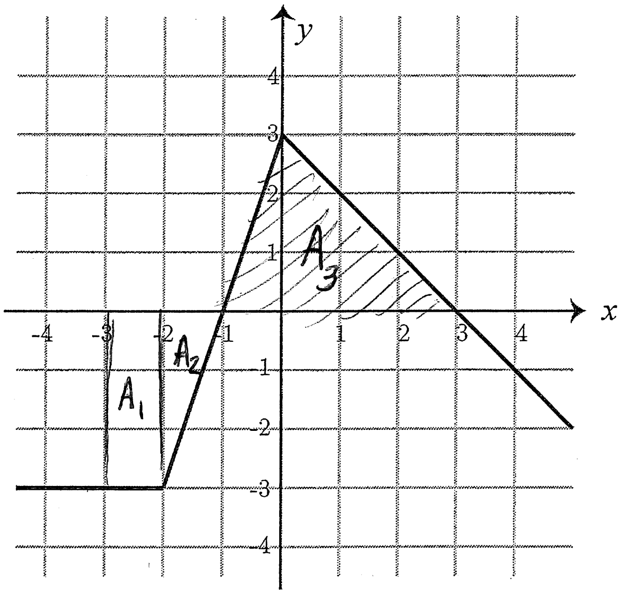
$= \frac{1}{2}(4)(3) - (1)(3) - \frac{1}{2}(1)(3)$

$= 6 - 3 - \frac{3}{2}$

$= \boxed{1.5}$

Possibilities:

(a) 2.5
(b) 7.5
(c) 21.5
(d) 6
[✓ (e) 1.5]



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15. Suppose that $\int_4^{13} f(x)\,dx = 20$, $\int_{28}^{45} f(x)\,dx = 48$, and $\int_4^{45} f(x)\,dx = 12$. Find the value of $\int_{13}^{28} f(x)\,dx$.

$$\int_{13}^{28} f(x)dx = \int_{4}^{45} f(x)dx - \int_{4}^{13} f(x)dx - \int_{28}^{45} f(x)dx$$

$$= 12 - 20 - 48$$

$$= \boxed{-56}$$

Possibilities:

(a) 80

[✓ (b) −56]

(c) −80

(d) 16

(e) −504

---

16. Suppose that $\int_5^{20} f(x)\,dx = 14$. Find the value of $\int_5^{20} (3f(x) + 60)\,dx$.

Possibilities:

(a) 102

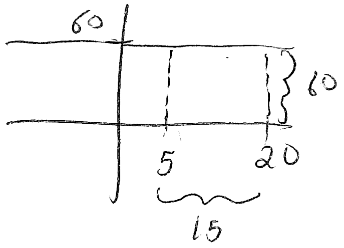
(b) 57

[✓ (c) 942]

(d) 222

(e) 1242

$$= 3\int_{5}^{20} f(x)dx + \int_{5}^{20} 60\,dx$$



$$= 3(14) + (15)(60)$$

$$= 42 + 900$$

$$= \boxed{942}$$

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17. Find the **average value** of $f(x)$ on the interval [7, 15] given that $f(x) = \begin{cases} 40 & \text{if } x < 10 \\ -80 & \text{if } x \geq 10. \end{cases}$

Possibilities:

(a) −15

(b) −140

(c) −20

[✓ (d) −35]

(e) 6

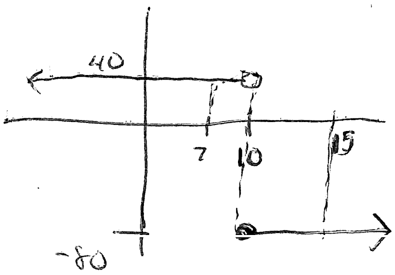
Ave value $= \dfrac{\int_7^{15} f(x)dx}{15-7}$

$= \dfrac{\int_7^{10} f(x)dx + \int_{10}^{15} f(x)dx}{8}$

$= \dfrac{40(3) - 80(5)}{8}$

$= \dfrac{120 - 400}{8}$

$= \dfrac{-280}{8} = \boxed{-35}$



18. Estimate the area under the graph of $y = -x^2 + 50x$ for x between 5 and 17, by using a partition that consists of 4 equal subintervals of $[5, 17]$ and using the **right** endpoint of each subinterval as a sample point.

Possibilities:

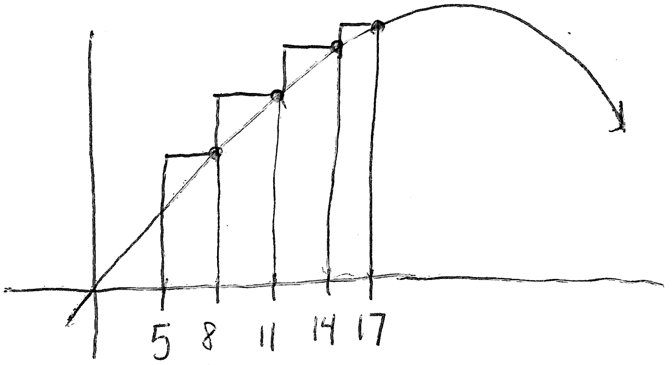
[✓ (a) 5490]

(b) 1830

(c) 4482

(d) 6165

(e) 5004



$-(8)^2+50(8)=336$

$-(11)^2+50(11)=429$

$-(14)^2+50(14)=$ 504

$-(17)^2+50(17)=561$

Rect. #	base	height	Area
1	3	336	1008
2	3	429	1287
3	3	504	1512
4	3	561	+ 1683
			5490 ← Total Area

19. Suppose you estimate the area under the graph of $f(x) = x^3$ from $x = 7$ to $x = 25$ by adding the areas of the rectangles as follows: partition the interval into 6 equal subintervals and use the right endpoint of each interval to determine the height of the rectangle. What is the area of the 5 th rectangle?

Possibilities:

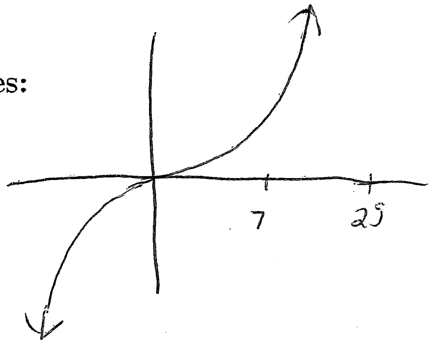
(a) 10648

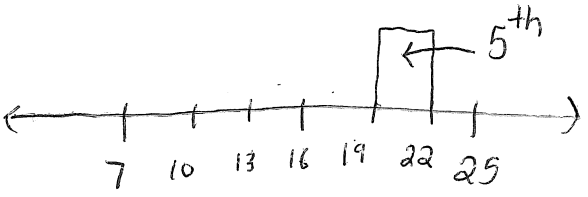
(b) 121275

[✓ (c) 31944]

(d) 20577

(e) $\frac{103935}{4}$





$\frac{25-7}{6} = \frac{18}{6} = 3$ ↑ base of each rectangle

Area of 5th rectangle
$= (\text{base})(\text{height})$
$= 3f(22)$
$= 3(22)^3$
$= \boxed{31944}$

20. Suppose you are given the following data points for a function $f(x)$.

x	1	2	3	4
$f(x)$	6	12	17	18

If f is a **linear** function on each interval between the given points, find $\int_1^4 f(x)\,dx$.

Possibilities:

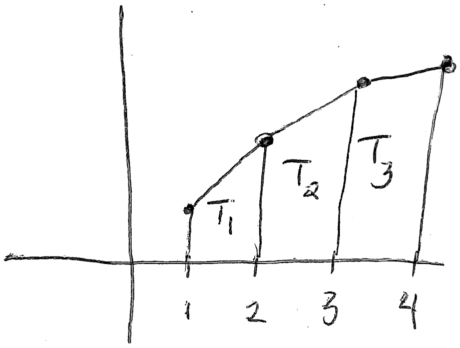
(a) 47

(b) 53

(c) 35

(d) 153

[✓ (e) 41]



$\parallel$

Area of 3 trapezoids $= T_1 + T_2 + T_3$
$= (1)\left(\frac{6+12}{2}\right) + (1)\left(\frac{12+17}{2}\right) + (1)\left(\frac{17+18}{2}\right)$
$= \frac{18}{2} + \frac{29}{2} + \frac{35}{2}$
$= \frac{82}{2}$
$= \boxed{41}$

Some Formulas

1. **Areas:**
    - (a) Triangle $A = \frac{bh}{2}$
    - (b) Circle $A = \pi r^2$
    - (c) Rectangle $A = lw$
    - (d) Trapezoid $A = \frac{h_1 + h_2}{2} b$
2. **Volumes:**
    - (a) Rectangular Solid $V = lwh$
    - (b) Sphere $V = \frac{4}{3}\pi r^3$
    - (c) Cylinder $V = \pi r^2 h$
    - (d) Cone $V = \frac{1}{3}\pi r^2 h$