MA123 — Elem. Calculus Fall 2018
Exam 3 2018-11-15

Name: SOLUTIONS **Sec.:** Revised

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name and section number on this page.

GOOD LUCK!

3.	(a)	(b)	(c)	(d)	(e)	**12.**	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)	**13.**	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)	**14.**	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)	**15.**	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)	**16.**	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)	**17.**	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)	**18.**	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)	**19.**	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)	**20.**	(a)	(b)	(c)	(d)	(e)

For grading use:

Multiple Choice		Short Answer
(number right)	(5 points each)	(out of 10 points)

Total	
	(out of 100 points)

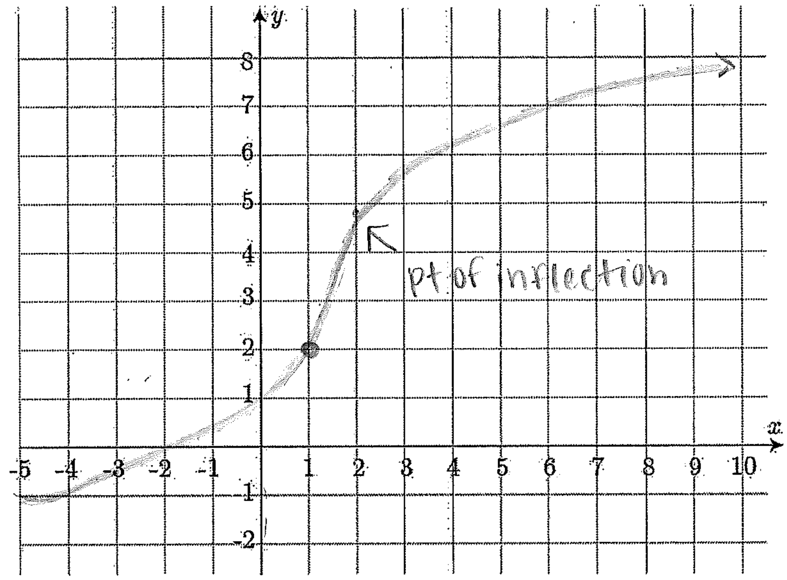
Fall 2018 Exam 3 Short Answer Questions

Write your answers on this page.

1. Sketch the graph of a **continuous** function $y = f(x)$ which satisfies the following:

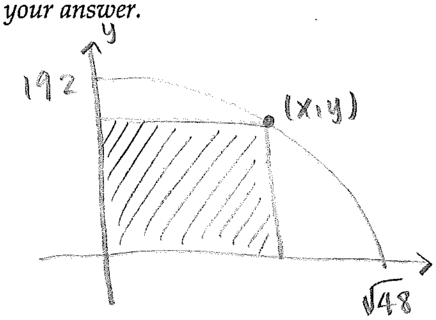
   $f(1) = 2$; $\underbrace{f'(x) \geq 0 \text{ for all } x}$; f is concave up for $x < 2$ and concave down for $x > 2$.

always increasing



2. Find the area of the largest rectangle with one corner at the origin, the opposite corner in the first quadrant on the graph of the parabola $f(x) = 192 - 4x^2$, and sides parallel to the axes.

   You must ***CLEARLY USE CALCULUS*** *to find and justify your answer.*



$A = xy$

$= x(192 - 4x^2)$

$= 192x - 4x^3$

$A' = 192 - 12x^2$

$\Rightarrow A' = 0$ when

$192 - 12x^2 = 0$

$-12x^2 = -192$

$x^2 = 16$

$x = 4, -4$

$0 = 192 - 4x^2$

$-192 = -4x^2$

$48 = x^2 \Rightarrow x = \sqrt{48}, -\sqrt{48}$

Test interval:

A: ↗ ↘

A': + −

0 test 1 4 test 100

$A(0) = 0$

$A(4) = 192(4) - 4(4)^3$

$= 768 - 256 = 512$

Largest Possible Area: 512

Name: ______________________

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

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3. Where is the function $f(t) = t^3 - 6t^2 - 63t + 5$ decreasing?

Possibilities:

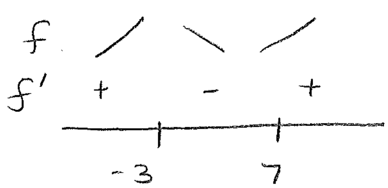
$\boxed{\text{(a) } -3 < t < 7}$

(b) $f(t)$ is always decreasing

(c) $t < -3$ and $t > 7$

(d) $t < 2$

(e) $t > 2$



decreasing $\Rightarrow f'(x) < 0$

$f(x) = t^3 - 6t^2 - 63t + 5$

$f'(x) = 3t^2 - 12t - 63$

$3t^2 - 12t - 63 < 0$

$3(t^2 - 4t - 21) < 0$

$t^2 - 4t - 21 < 0$

$(t-7)(t+3) < 0$

$\Rightarrow \boxed{-3 < t < 7}$

---

4. Where is the function $f(t) = t^3 - 6t^2 - 63t + 5$ concave up?

Possibilities:

(a) $-3 < t < 7$

$\boxed{\text{(b) } t > 2}$

(c) $f(t)$ is always concave up

(d) $t < 2$

(e) $t < -3$ and $t > 7$

concave up $\Rightarrow f''(x) > 0$

$f(x) = t^3 - 6t^2 - 63t + 5$

$f'(x) = 3t^2 - 12t - 63$

$f''(x) = 6t - 12$

$6t - 12 > 0$

$6t > 12$

$\boxed{t > 2}$

---

5. Suppose the derivative of $g(t)$ is $g'(t) = 4(t-8)(t-9)(t-5)$. For t in which interval(s) is g increasing?

Possibilities:

(a) $(\frac{22}{3} - \frac{1}{3}\sqrt{13}, \frac{22}{3} + \frac{1}{3}\sqrt{13})$

(b) $(-\infty, 5) \cup (8, 9)$

(c) $\boxed{(5, 8) \cup (9, \infty)}$

(d) $(-\infty, \frac{22}{3} - \frac{1}{3}\sqrt{13}) \cup (\frac{22}{3} + \frac{1}{3}\sqrt{13}, \infty)$

(e) $(4, 5) \cup (8, 9)$

Increasing $\Rightarrow g'(t) > 0$

$g'(t) = 4(t-8)(t-9)(t-5) > 0$

$t-8 > 0 \Rightarrow t > 8$

$t-9 > 0 \Rightarrow t > 9$

$t-5 > 0 \Rightarrow t > 5$

Test intervals:

0	5	6	8	8.5	9	100
−		+		−		+

$\boxed{(5,8) \cup (9,\infty)}$

6. Suppose the derivative of $g(t)$ is $g'(t) = 13(t-4)(t-8)$. For t in which interval(s) is g concave up?

Possibilities:

(a) $(4, 8)$

(b) $(-\infty, 6)$

(c) $(-\infty, 4) \cup (8, \infty)$

(d) $(4, 6) \cup (8, 13)$

(e) $\boxed{(6, \infty)}$

concave up $\Rightarrow g''(t) > 0$

$g'(t) = 13(t-4)(t-8)$

$= 13(t^2 - 12t + 32)$

$g''(t) = 13(2t-12)$

$= 26(t-6)$

$t-6 > 0 \Rightarrow t > 6$

Test intervals:

0	6	100
−		+

$\boxed{(6,\infty)}$

7. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$. Where is the original function $f(x)$ decreasing?

Possibilities:

(a) $(-\infty, -1)$

(b) $(-2, \infty)$

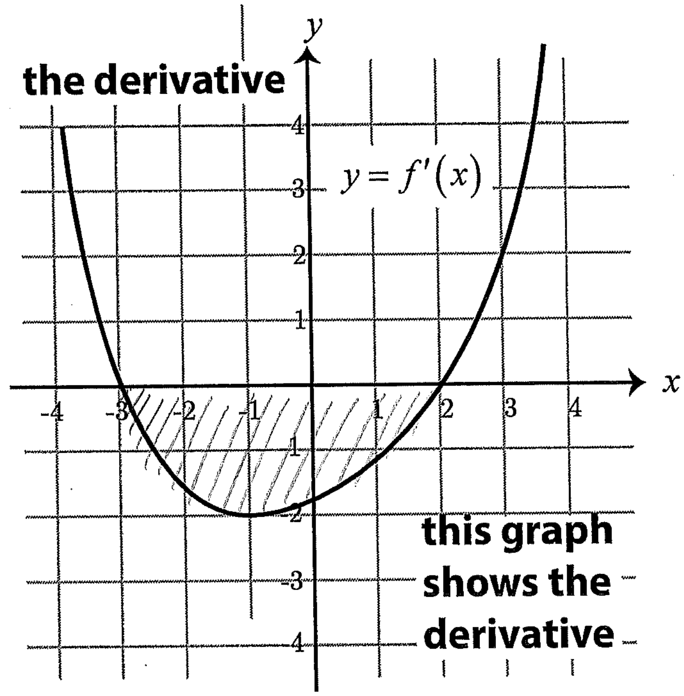
(c) $(-3, 2)$

(d) $(-\infty, -3)$ and $(2, \infty)$

(e) $(-1, \infty)$

Decreasing ⇒ $f'(x) < 0$

x interval where y value fall below the x-axis : $(-3, 2)$.



8. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$. Where is the original function $f(x)$ concave up?

Possibilities:

(a) $(-1, \infty)$

(b) $(-2, \infty)$

(c) $(-\infty, -1)$

(d) $(-\infty, -3)$ and $(2, \infty)$

(e) $(-3, 2)$

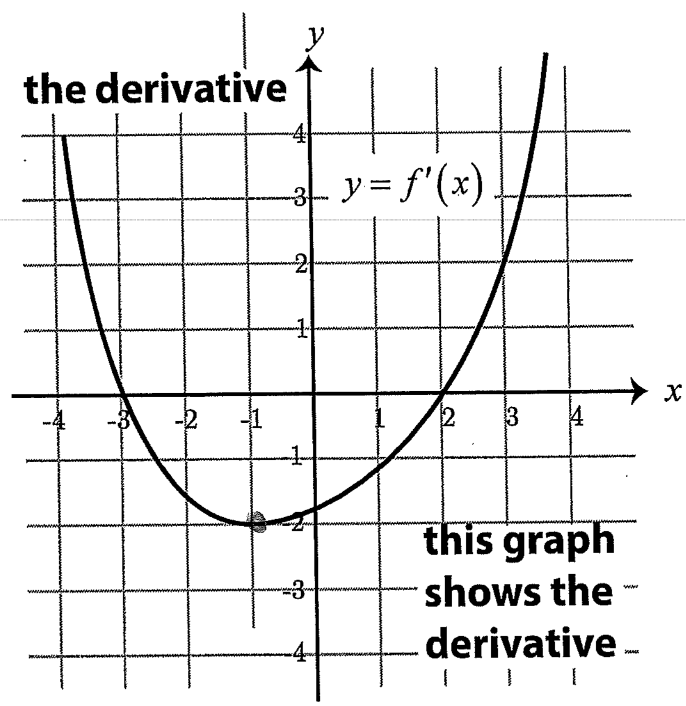
concave up ⇒ $f''(x) > 0$

$f'(x)$ changes direction at $x = -1$.

$f'(x)$ falling ⇒ concave down

$f'(x)$ rising ⇒ concave up

$f'(x)$ rising at $(-1, \infty)$



9. Find the critical numbers of the function

$$f(x) = \frac{4x}{8x^2 + 648}.$$

Possibilities:

(a) $-\frac{4}{8}, \frac{4}{648}$

(b) $-81, 4$

(c) $-9, 9$

(d) $-\sqrt{\frac{1}{9}}, \sqrt{\frac{1}{9}}$

(e) $-81, 0$

$f'(x) = \frac{(8x^2+648)\cdot 4 - 4x(16x)}{(8x^2+648)^2}$

$f'(x) = 0 \Rightarrow$ critical numbers!

$0 = \frac{(8x^2+648)4 - 4x(16x)}{(8x^2+648)^2}$

$= \frac{32x^2+2592-64x^2}{(8x^2+648)^2}$ * multiply both sides by denominator

$= 32x^2+2592-64x^2$

$= -32x^2+2592$

$-2592 = -32x^2$

$81 = x^2 \Rightarrow \boxed{x = 9, -9}$

10. Consider the graph of the original function, $f(x)$.
For this function, what are the signs of $f'(-3)$ and $f''(-3)$?

Possibilities:

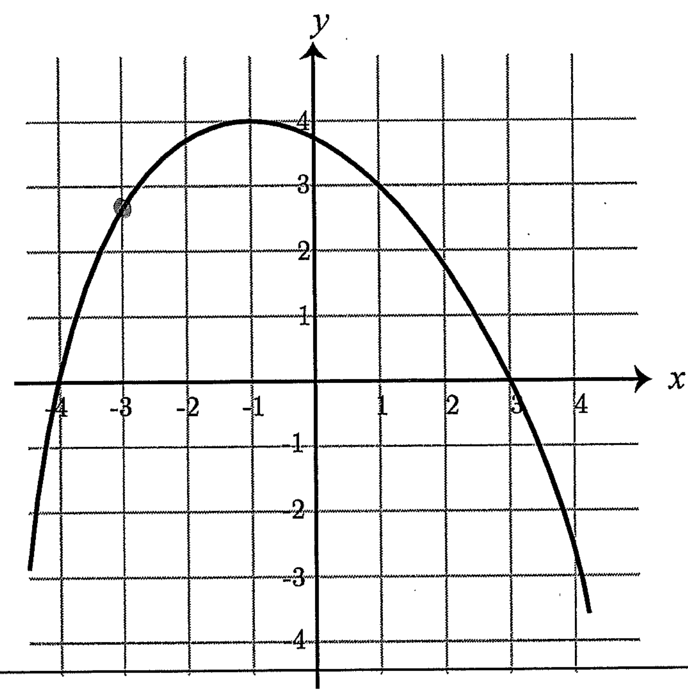
(a) $f'(-3) > 0$ and $f''(-3) < 0$

(b) $f'(-3) = 0$ and $f''(-3) < 0$

(c) $f'(-3) > 0$ and $f''(-3) > 0$

(d) $f'(-3) < 0$ and $f''(-3) < 0$

(e) $f'(-3) < 0$ and $f''(-3) > 0$

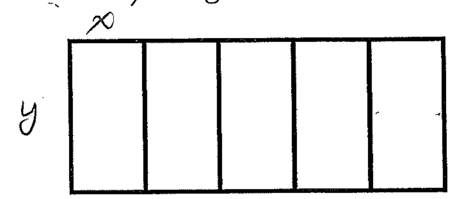


$f(-3) = +$

$f'(-3) = +$ (increasing)

$f''(-3) = -$ (concave down)

11. A farmer builds a rectangular pen with 5 vertical partitions (6 vertical sides) using 600 feet of fencing. What is the maximum possible total area of the pen?



Possibilities:

(a) 22500

(b) 15000

(c) 600

$\boxed{\text{(d) } 7500}$

(e) $\frac{45000}{7}$

each length = x

total height = y

$600 = 2x + 6y$ (perimeter)

$A = xy$ (area)

Solve for y: $600 = 2x + 6y$

$600 - 2x = 6y$

$y = 100 - \frac{1}{3}x$

Plug in to area formula:

$A = x(100 - \frac{1}{3}x)$

Find derivative of A:

$A' = x(-\frac{1}{3}) + 1 \cdot (100 - \frac{1}{3}x)$

$= -\frac{1}{3}x + 100 - \frac{1}{3}x = -\frac{2}{3}x + 100$

Find critical pts:

$A' = 0$

$\Rightarrow -\frac{2}{3}x + 100 = 0$

$-\frac{2}{3}x = -100$

$x = 150$

If $x < 150$, $A' > 0$

If $x > 150$, $A' < 0$

So area is maximized at $x = 150$!

$600 = 2(150) + 6y$

$\Rightarrow y = 50$

$A = (150)(50) = \boxed{1500}$

12. A landscape architect wishes to enclose a rectangular garden on one side by a brick wall costing \$70 per foot, and on the other three sides by a metal fence costing \$30 per foot. If the area of the garden is 300 square feet, find the lowest possible cost to enclose the garden.

Possibilities:

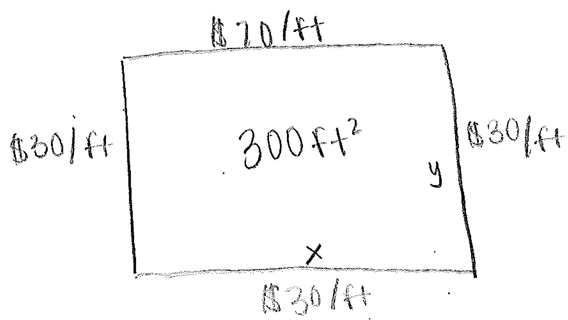
(a) \$2684.78

(b) \$2684.28

(c) \$2682.78

(d) \$2683.78

$\boxed{\text{(e) } \$2683.28}$



$C = 70x + 30x + 30y + 30y$

$= 100x + 60y$

$A = 300 = xy$

$\Rightarrow y = 300/x$ plug into C!

$C = 100x + 60(300/x)$

$= 100x + 18000/x$

$C' = 100 + (-18000x^{-2})$

$C' = 0 \Rightarrow 0 = 100 - 18000x^{-2}$

$-100 = -18000x^{-2}$

$100x^2 = 18000$

$x^2 = 180$

$x \approx 13.4164$

Test interval:

13.4164

C'(10) − | C'(20) +

minimum

$y = 300/x = 300/13.4164 \approx 22.36$

$C = 100(13.4164) + 60(22.36)$

$\boxed{= 2683.24}$

Rounding caused small difference!

13. Given the function $f(x) = \begin{cases} x & \text{if } x < 26 \\ 26 & \text{if } x \geq 26 \end{cases}$

evaluate the definite integral

$$\int_0^{36} f(x)\,dx$$

Possibilities:

(a) 597

(b) 598

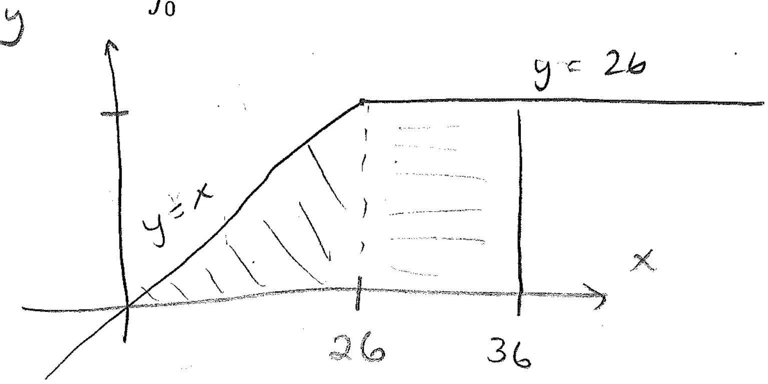
(c) 599

(d) 600

(e) 601

graph

$y = f(x)$



triangle rectangle

$$\int_0^{36} f(x)dx = \int_0^{26} f(x)\,dx + \int_{26}^{36} f(x)\,dx$$

$$= \underbrace{\tfrac{1}{2}(26)(26)}_{\frac{1}{2}\text{ base height}} + \underbrace{10(26)}_{\text{base height}} = 338 + 260 = 598$$

14. The graph of $y = f(x)$ shown below includes a semicircle and a straight line. Evaluate the definite integral $\int_{-2}^{4} f(x)\,dx$.

Possibilities:

(a) $-2\pi - 8$

(b) $-2\pi + 8$

(c) $\pi + 8$

(d) $2\pi + 8$

(e) $-\pi + 8$

$(-2,0)$: $\frac{\pi r^2}{4} = \frac{\pi(2)^2}{4}$

$= \pi$ below x-axis $\Rightarrow -\pi$

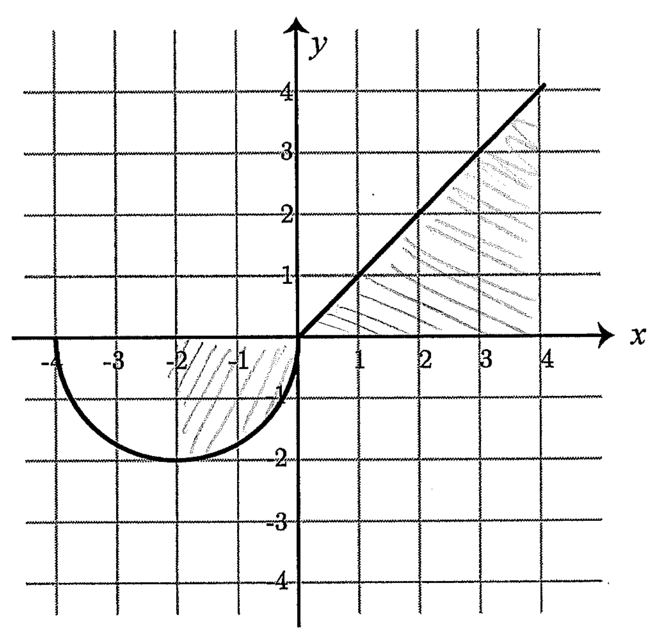
$(0,4)$: $\frac{1}{2}bh$

$= \frac{1}{2}(4)(4)$

$= 8$

$(-2,0) + (0,4)$

$\boxed{-\pi + 8}$



15. Suppose that $\int_3^{37} f(x)\,dx = 13$, $\int_3^{12} f(x)\,dx = 27$, and $\int_{18}^{37} f(x)\,dx = 45$. Find the value of $\int_{12}^{18} f(x)\,dx$.

Possibilities:

(a) 85

(b) 5

(c) −656

(d) −59

(e) −85

$$\int_3^{37} f(x)dx = \int_3^{12} f(x)dx + \int_{12}^{18} f(x)dx + \int_{18}^{37} f(x)dx$$

$13 = 27 + x + 45$

$13 = 72 + x$

$13 - 72 = \boxed{-59 = x}$

16. Suppose that $\int_4^{17} f(x)\,dx = 13$. Find the value of $\int_4^{17} (3f(x) + 6)\,dx$.

Possibilities:

(a) 57

(b) 45

(c) 117

(d) 141

(e) 52

$$\int_4^{17} (3f(x) + 6)\,dx$$

$$= \int_4^{17} 3f(x)\,dx + \int_4^{17} 6\,dx$$

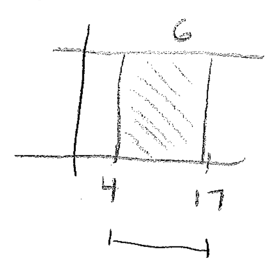
$$= 3\int_4^{17} f(x)\,dx + 6(13)$$

$= 3 \cdot 13 + 78$

$= 39 + 78$

$= \boxed{117}$

$y=6$



17. Find the **average value** of $f(x)$ on the interval [7, 15] given that $f(x) = \begin{cases} 30 & \text{if } x < 10 \\ -70 & \text{if } x \geq 10. \end{cases}$

Possibilities:

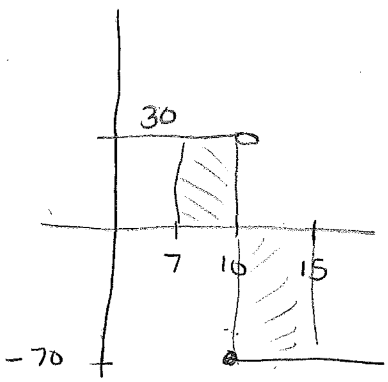
$\boxed{\text{(a) } -\frac{65}{2}}$

(b) 6

(c) -20

(d) -130

(e) $-\frac{25}{2}$



Average value $= \frac{1}{b-a}\int_a^b f(x)\,dx$

$= \frac{1}{15-7}\int_7^{15} f(x)\,dx$

$= \frac{1}{8}\left(\int_7^{10} 30\,dx + \int_{10}^{15} -70\,dx\right)$

$= \frac{1}{8}\left(30(3) + (-70)(5)\right)$

$= \frac{1}{8}(90 - 350)$

$= \frac{1}{8}(-260) = \frac{-260}{8}$

$= \boxed{\frac{-65}{2}}$

18. Estimate the area under the graph of $y = -x^2 + 30x$ for x between 2 and 10, by using a partition that consists of 4 equal subintervals of $[2, 10]$ and use the left endpoint of each subinterval as a sample point.

Possibilities:

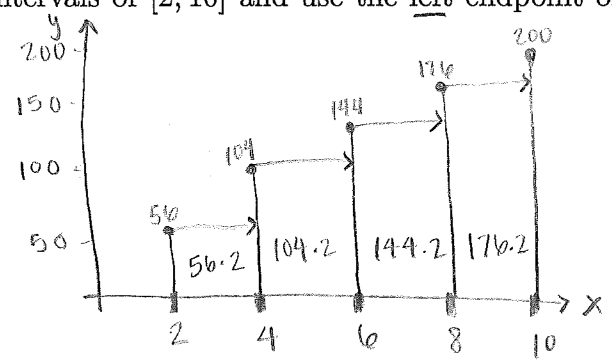
(a) $\frac{3328}{3}$

$\boxed{\text{(b) } 960}$

(c) 1248

(d) 624

(e) 1360



$2(56+104+144+176)$

$= 2(480)$

$= \boxed{960}$

19. Suppose you estimate the area under the graph of $f(x) = \dfrac{1}{x}$ from $x = 6$ to $x = 24$ by adding the areas of the rectangles as follows: partition the interval into 6 equal subintervals and use the right endpoint of each interval to determine the height of the rectangle. What is the area of the 4th rectangle?

Possibilities:

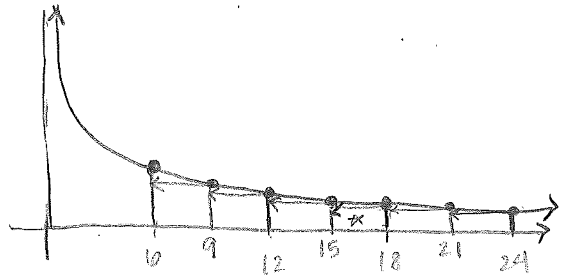
(a) $\frac{341}{280}$

(b) $\frac{1}{5}$

(c) $\boxed{\frac{1}{6}}$

(d) $\ln(3) - \ln(5) + \ln(2)$

(e) $\frac{1}{18}$



$(18-15)\cdot\frac{1}{18}$

$= 3\cdot\frac{1}{18}$

$= \boxed{\frac{1}{6}}$

20. A train travels in a straight westward direction along a track. The speed of the train varies, but it is measured at regular time intervals of 1/10 hour. The measurements for the first half hour are:

time	0	.1	.2	.3	.4	.5
speed	0	5	10	15	18	27

Estimate the total distance (in miles) traveled by the train during the first half hour by assuming the speed is a linear function of t on the subintervals. The speed in the table is given in miles per hour. Use all six speed measurements in your estimate.

Possibilities:

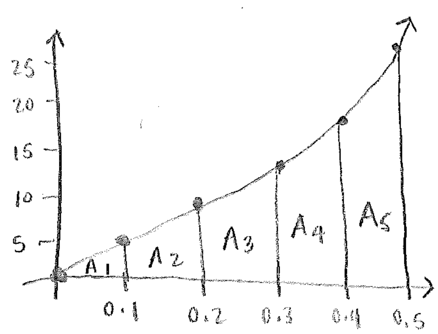
(a) 13.50 miles

(b) 6.00 miles

(c) 7.50 miles

(d) 2.50 miles

(e) $\boxed{\text{6.15 miles}}$



use trapezoids

$A_1 = \frac{(.1-0)(5+0)}{2} = 0.25$

$A_2 = \frac{(.2-.1)(10+5)}{2} = 0.75$

$A_3 = \frac{(.3-.2)(15+10)}{2} = 1.25$

$A_4 = \frac{(.4-.3)(18+15)}{2} = 1.65$

$A_5 = \frac{(.5-.4)(27+18)}{2} = 2.25$

$\boxed{\text{Sum} = 6.15}$

(or: find the average of the left- and right sums.)

Some Formulas

1. **Areas:**
   (a) Triangle $A = \frac{bh}{2}$
   (b) Circle $A = \pi r^2$
   (c) Rectangle $A = lw$
   (d) Trapezoid $A = \frac{h_1 + h_2}{2} b$

2. **Volumes:**
   (a) Rectangular Solid $V = lwh$
   (b) Sphere $V = \frac{4}{3}\pi r^3$
   (c) Cylinder $V = \pi r^2 h$
   (d) Cone $V = \frac{1}{3}\pi r^2 h$