MA123 — Elem. Calculus Spring 2017
Exam 3 2017-04-13

Name: SOLUTIONS Sec.: _____

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) b c d e

You have two hours to do this exam. Please write your name on this page, and at the top of page three.

GOOD LUCK!

3.	a	b	**c**	d	e	12.	**a**	b	c	d	e
4.	a	b	c	**d**	e	13.	a	b	c	d	**e**
5.	a	b	c	d	**e**	14.	a	b	**c**	d	e
6.	a	b	**c**	d	e	15.	a	**b**	c	d	e
7.	a	**b**	c	d	e	16.	a	b	c	**d**	e
8.	a	b	c	d	**e**	17.	a	**b**	c	d	e
9.	a	b	c	**d**	e	18.	a	**b**	c	d	e
10.	a	**b**	c	d	e	19.	**a**	b	c	d	e
11.	a	b	c	**d**	e	20.	a	b	c	**d**	e

For grading use:

Multiple Choice		Short Answer
(number right)	(5 points each)	(out of 10 points)

Total	
	(total 100 points)

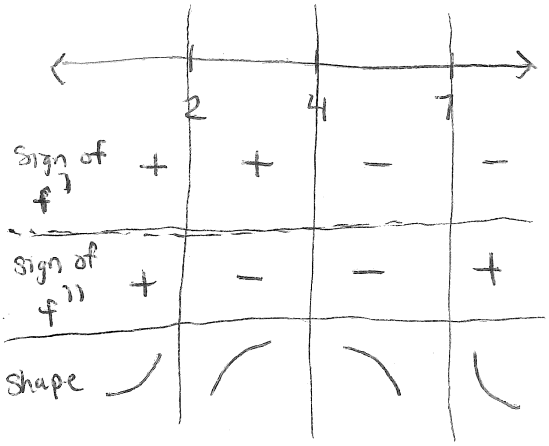
Name: SOLUTIONS Last 4 digits of Student ID:

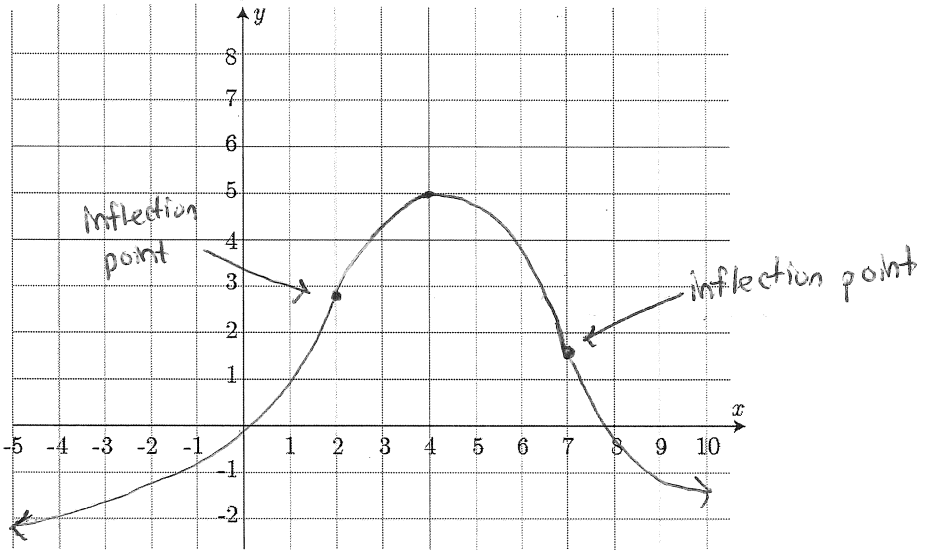
Spring 2017 Exam 3 Short Answer Questions

Write answers on this page. Your work must be clear and legible to be sure you will get full credit.

1. Sketch the graph of a **continuous** function $y=f(x)$ for which f is increasing on $(-\infty,4)$, decreasing on $(4,\infty)$, $f''(x)>0$ on $(-\infty,2)$ and $(7,\infty)$; $f''(x)<0$ on $(2,7)$.

$f'(x)<0$ ← decreasing → $f'(x)>0$





2. Suppose we know two nonnegative numbers x and y satisfying $2x+y=13$. Find the maximum possible value of their product xy. *You must **clearly use calculus** to find **and justify** your answer. Your final answer does **not** need to be simplified.*

Let $P=xy$

Get P in terms of one variable by using $2x+y=13$ to substitute

$y=13-2x$

$P=x(13-2x)=13x-2x^2$

Take derivative and find crit. points

$P'=13-4x$

$0=13-4x$

$4x=13$

$x=\frac{13}{4}$

P' is never undefined

Test

Method 1: Use closed interval

x is in $[0,\frac{13}{2}]$

$P(0)=0$

$P(\frac{13}{2})=0$

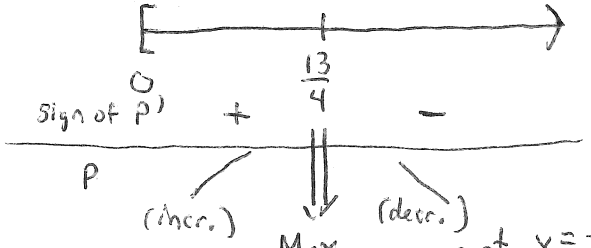
$P(\frac{13}{4})=\frac{13}{4}\left(13-2(\frac{13}{4})\right)$

$=\frac{13}{4}(\frac{13}{2})=\frac{169}{8}$ ← maximum value

Method 2: Sign chart for P'

$P'(1)=13-4(1)=9$

$P'(10)=13-4(10)=-27$



Max. occurs at $x=\frac{13}{4}$ since P changes from incr. to decr.

Maximum possible product: $\boxed{\frac{169}{8}}$

We would plug this into P to get the maximum value of product

Name: SOLUTIONS

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

3. Where is the function $f(t) = t^3 + 6t^2 - 36t + 8$ decreasing?

Possibilities:

(a) $t < -2$

(b) $f(t)$ is always decreasing

(c) $-6 < t < 2$

(d) $t > -2$

(e) $t < -6$ and $t > 2$

Find crit. pts.

$f'(t) = 3t^2 + 12t - 36$

f' is never undefined

Set $f'(t) = 0$

$0 = 3t^2 + 12t - 36$ (Divide by 3)

$0 = t^2 + 4t - 12$

$0 = (t+6)(t-2)$

$t = -6, 2$

$f'(-7) = (-7+6)(-7-2)$
$= (-1)(-9)$
$= 9$

$f'(0) = (6)(-2)$
$= -12$

$f'(3) = (3+6)(3-2)$
$= (9)(1)$
$= 9$

f is decr. on $(-6, 2)$

	$t=-7$	-6	$t=0$	2	$t=3$
sign of f'	+		−		+
f	incr.		decr.		incr.

4. Where is the function $f(t) = \dfrac{1}{t-51}$ concave up?

Possibilities:

(a) $f(t)$ is never concave up

(b) $-1 < t < 51$

(c) $t < 51$

(d) $t > 51$

(e) $f(t)$ is always concave up except at t = 51

$f(t) = (t-51)^{-1}$

$f'(t) = -(t-51)^{-2}$

$f''(t) = 2(t-51)^{-3} = \dfrac{2}{(t-51)^3}$

Find where $f''(t)$ is 0 or undefined.

It is undefined at $t = 51$.

$0 = \dfrac{2}{(t-51)^3}$ (Multiply both sides by denom.)

$0 = 2$

No solution ☹

$f''(50) = \dfrac{2}{-1} = -2$

$f''(52) = \dfrac{2}{1} = 2$

Sign of f''

	−	51	+
	$t=50$		$t=52$
f	conc. down		conc. up

f is conc. up when $t > 51$

5. Suppose the derivative of $g(t)$ is $g'(t) = 11(t-3)^2(t-7)$. For t in which interval(s) is g increasing?

Possibilities:

(a) $(-\infty, 3) \cup (7, \infty)$

(b) $(-\infty, 7)$

(c) $(3, 7)$

(d) $(3, 7) \cup (11, \infty)$

(e) $(7, \infty)$

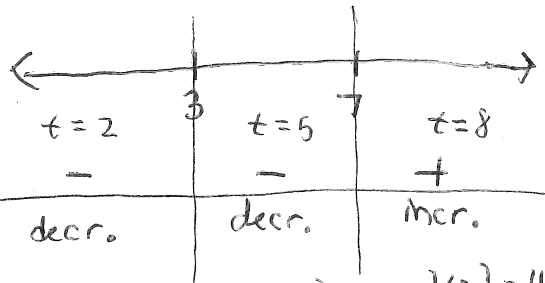
Set $g'(t) = 0$

$0 = 11(t-3)^2(t-7)$

$t = 3, 7$

g' is never undefined

g is increasing when $g'(t) > 0$



$g'(2) = 11(-1)^2(-5) = (+)(+)(-) = -$

$g'(5) = 11(2)^2(-2) = (+)(+)(-) = -$

$g'(8) = 11(5)^2(1) = (+)(+)(+) = +$

g is increasing on $(7, \infty)$

---

6. Suppose the derivative of $g(t)$ is $g'(t) = 81 - t^2$. Where is the function $g(t)$ concave up?

Possibilities:

(a) $f(t)$ is always concave up

(b) $t < -9$ and $t > 9$

(c) $t < 0$

(d) $-9 < t < 9$

(e) $t > 0$

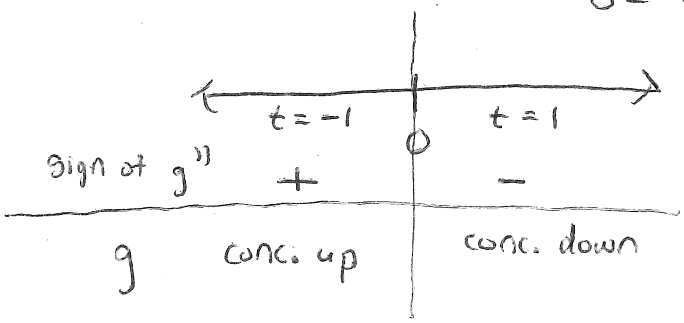
$g''(t) = -2t$

g'' is never undefined

Set $g''(t) = 0$

$0 = -2t$

$0 = t$



$g''(-1) = 2$ $\quad g''(1) = -2$ $\quad \Rightarrow g$ is conc. up on $(-\infty, 0)$

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7. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$.
Where is the original function $f(x)$ decreasing?

f is decreasing when f' is negative

Possibilities:

(a) nowhere

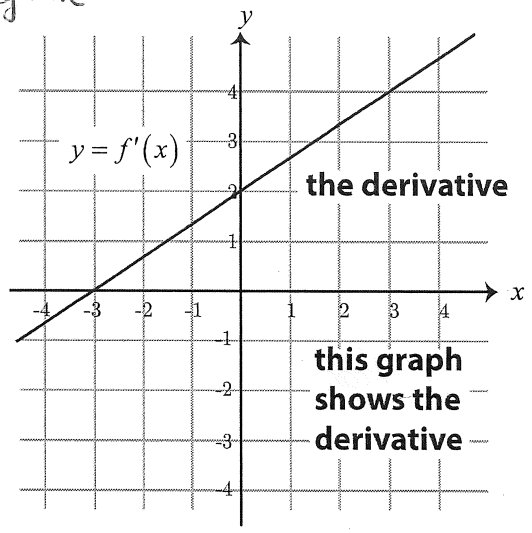
This happens on $(-\infty, -3)$

(b) $(-\infty, -3)$

(c) everywhere

(d) $(2, \infty)$

(e) $(-3, \infty)$



8. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$.
Where is the original function $f(x)$ concave up?

Possibilities:

(a) nowhere

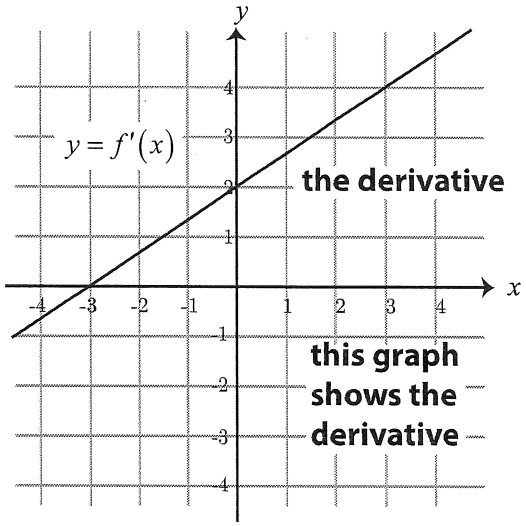
(b) $(2, \infty)$

(c) $(-\infty, -3)$

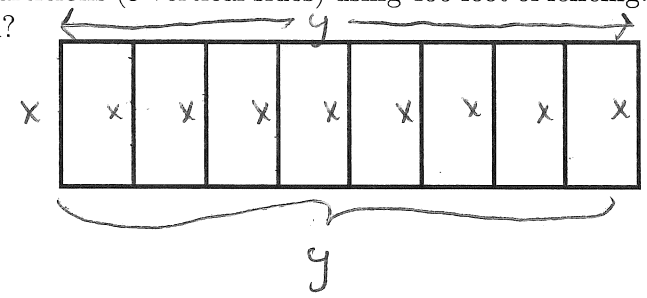
(d) $(-3, \infty)$

(e) everywhere

f is concave up when f' is increasing which happens everywhere



9. A farmer builds a rectangular pen with 8 vertical partitions (9 vertical sides) using 400 feet of fencing. What is the maximum possible total area of the pen?



Possibilities:

(a) 2000

(b) 400

(c) 10000

(d) $\frac{20000}{9}$

(e) $\frac{10000}{3}$

$A = xy$

$9x + 2y = 400$

$2y = 400 - 9x$

$y = 200 - \frac{9}{2}x$

Substitute this into A:

$A = x\left(200 - \frac{9}{2}x\right) = 200x - \frac{9}{2}x^2$

$A' = 200 - 9x$

A' is never undefined

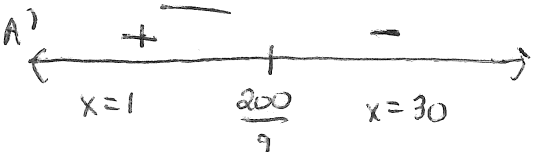
Set $A' = 0$

$\Rightarrow 0 = 200 - 9x$

$9x = 200$

$x = \frac{200}{9} \Rightarrow A = \frac{200}{9}\left(200 - \frac{9}{2}\left(\frac{200}{9}\right)\right) = \boxed{\frac{20000}{9}}$

Test



$A'(1) = 191$ $\quad A'(30) = -70$

$\Rightarrow A$ has a max. at $x = \frac{200}{9}$

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10. A car rental agency rents 190 cars per day at a rate of \$27 dollars per day. For each 1 dollar increase in the daily rate, 3 fewer cars are rented. At what rate should the cars be rented to produce maximum income?

Possibilities:

(a) \$44.57 per day

(b) \$45.17 per day

(c) \$44.77 per day

(d) \$45.37 per day

(e) \$45.97 per day

$R = NP$

R = revenue

N = # of cars rented

P = rate per car

Find an equation involving P and N (linear equation)

Point: $(P, N) = (27, 190)$

Slope $= \frac{-3}{1} = -3$

equation: $N - 190 = -3(P - 27) = -3P + 81$

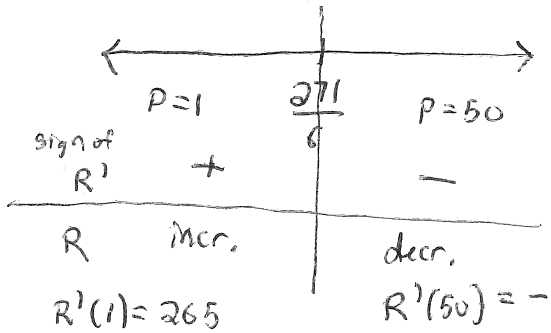
$N = -3P + 271$

$R = (-3P + 271)P = -3P^2 + 271P$

$R' = -6P + 271$ ← this is never undefined

$0 = -6P + 271 \Rightarrow 6P = 271 \Rightarrow P = \frac{271}{6} \approx 45.17$

Test



$R'(1) = 265$ $\quad R'(50) = -29$

$\Rightarrow R$ has a max. at $P = \frac{271}{6}$

11. Find the critical numbers of the function $f(x) = xe^{13x+3}$.

Possibilities:

(a) $-\frac{3}{13}, 0$

(b) 0

(c) $-\frac{1}{13}, 0, e^{13}$

(d) $-\frac{1}{13}$

(e) $-\frac{3}{13}$

Product Rule and Chain Rule

$f'(x) = x\left(e^{13x+3}\right)' + e^{13x+3}(x)'$

$f'(x) = x\,e^{13x+3}(13) + e^{13x+3}(1)$

f' is never undefined

Set $f'(x) = 0$

$0 = 13xe^{13x+3} + e^{13x+3}$

$0 = e^{13x+3}(13x+1)$

e^{13x+3} is always positive

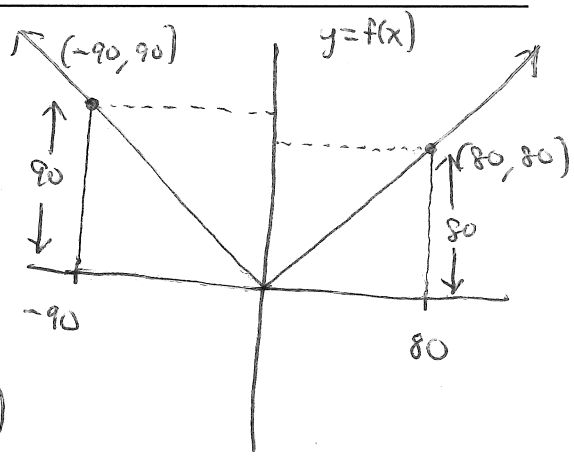
$\Rightarrow 13x+1 = 0$

$13x = -1 \Rightarrow x = -\frac{1}{13}$.

12. Given the function $f(x) = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

evaluate the definite integral

$$\int_{-90}^{80} f(x)\,dx$$



Possibilities:

(a) 7250

(b) 0

(c) 850

(d) 14450

(e) −850

= Signed area under curve

$= \frac{1}{2}(90)(90) + \frac{1}{2}(80)(80)$

$= 4050 + 3200$

$= 7250$

13. The graph of $y = f(x)$ shown below consists of straight lines. Evaluate the definite integral $\int_{-3}^{3} f(x)\,dx$.

$= A_3 - A_1 - A_2$

above axis — below axis

Possibilities:

(a) 7.5

(b) 2.5

(c) 6

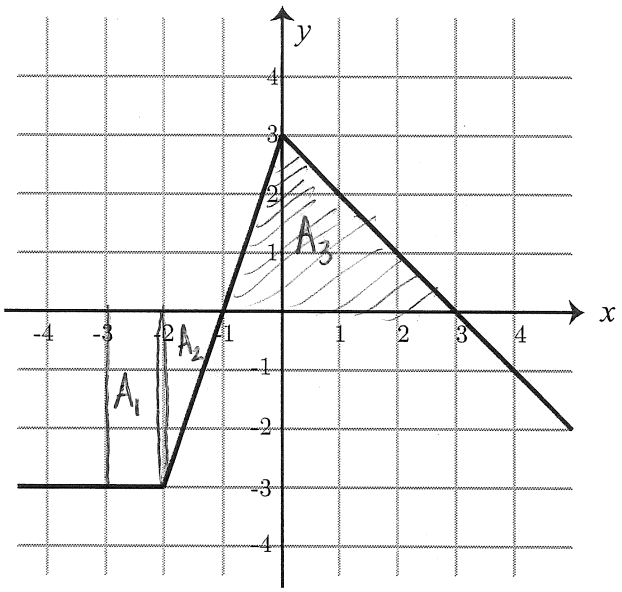
(d) 21.5

(e) 1.5

$= \frac{1}{2}(4)(3) - (1)(3) - \frac{1}{2}(1)(3)$

$= 6 - 3 - \frac{3}{2}$

$= 1.5$



14. The graph of $y = f(x)$ shown below includes semicircles and a straight line. Evaluate the definite integral $\int_{-4}^{2} f(x)\,dx$. Use $\pi = 3.14$.

$= A_2 - A_1$

Possibilities:

(a) 7.85

(b) −9.42

(c) −4.71

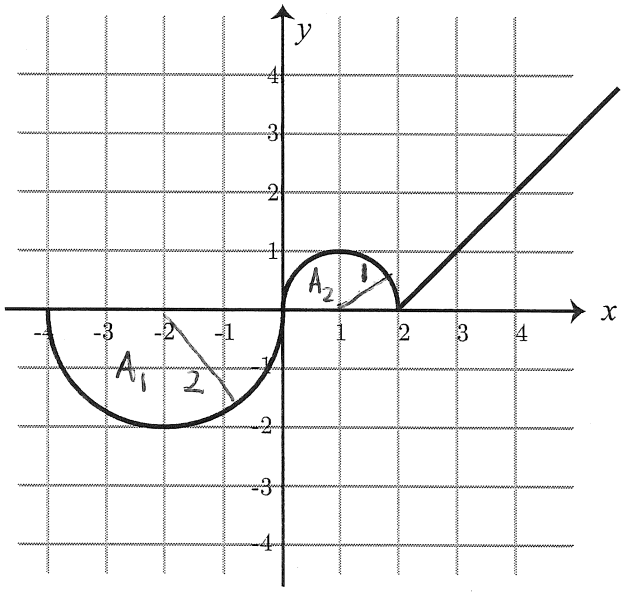
(d) −7.85

(e) 4.71

$= \frac{1}{2}\pi(1)^2 - \frac{1}{2}\pi(2)^2$

$= \frac{\pi}{2} - \frac{4\pi}{2}$

$= -\frac{3\pi}{2} \approx -4.71$



15. Suppose that $\int_{15}^{17} f(x)\,dx = 22$ and $\int_{6}^{17} f(x)\,dx = 7$. Find the value of $\int_{6}^{15} f(x)\,dx$.

Possibilities:

(a) 29

(b) -15

(c) $-\frac{5}{3}$

(d) -29

(e) 15

$$\int_6^{15} f(x)\,dx = \int_6^{17} f(x)\,dx - \int_{15}^{17} f(x)\,dx$$

$$= 7 - 22$$

$$= -15$$

16. Suppose that $\int_{7}^{23} f(x)\,dx = 9$ and $\int_{7}^{23} g(x)\,dx = 19$. Find the value of $\int_{7}^{23} (2f(x) + 4g(x))\,dx$.

Possibilities:

(a) 1504

(b) 8

(c) 544

(d) 94

(e) 96

$$= 2\int_7^{23} f(x)\,dx + 4\int_7^{23} g(x)\,dx$$

$$= 2(9) + 4(19)$$

$$= 18 + 76$$

$$= 94$$

17. Find the **average value** of $f(x)$ on the interval $[3, 11]$ given that $f(x) = \begin{cases} 90 & \text{if } x < 6 \\ -10 & \text{if } x \geq 6. \end{cases}$

Possibilities:

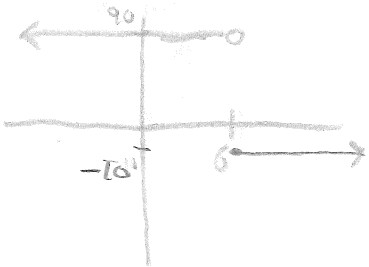
(a) $-\frac{25}{2}$

(b) $\frac{55}{2}$

(c) 6

(d) 110

(e) 40



$$\text{Ave value} = \frac{\int_3^{11} f(x)\,dx}{11-3}$$

$$= \frac{\int_3^{11} f(x)\,dx}{8} = \frac{\int_3^6 f(x)\,dx + \int_6^{11} f(x)\,dx}{8} = \frac{90(3) - 10(5)}{8}$$

$$= \frac{270-50}{8}$$

$$= \frac{220}{8}$$

$$= \frac{55}{2}$$

18. Estimate the area under the graph of $y = -x^2 + 30x$ for x between 2 and 14, by using a partition that consists of 4 equal subintervals of $[2, 14]$ and using the **right** endpoint of each subinterval as a sample point.

Possibilities:

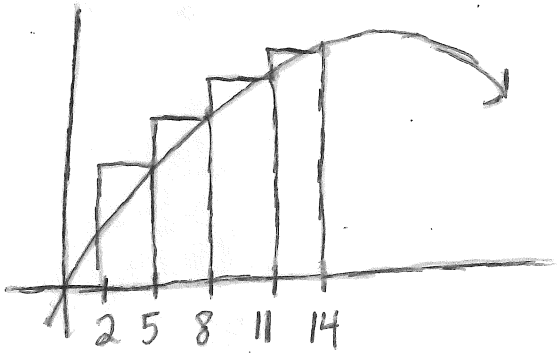
(a) 2370

(b) 2202

(c) 734

(d) 1698

(e) 1968



Rect #	base	height	Area
1	3	125	375
2	3	176	528
3	3	209	627
4	3	224	672
			2202

Total Area

19. Suppose you estimate the area under the graph of $f(x) = \dfrac{1}{x}$ from $x = 5$ to $x = 45$ by adding the areas of the rectangles as follows: partition the interval into 20 equal subintervals and use the right endpoint of each interval to determine the height of the rectangle. What is the area of the 10th rectangle?

Possibilities:

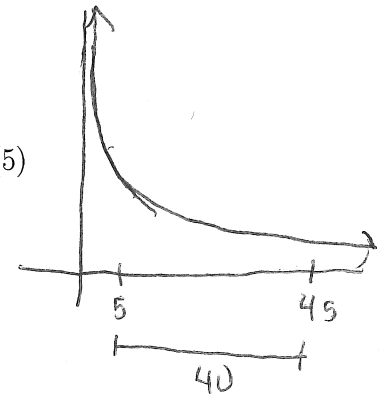
(a) $\frac{2}{25}$

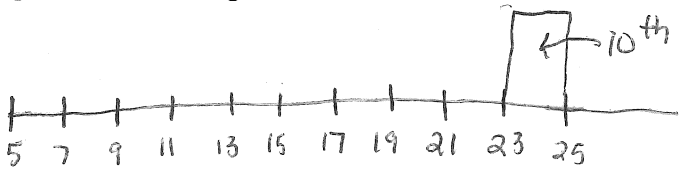
(b) $-\ln(23) + 2\ln(5)$

(c) 2.032416314

(d) $\frac{2}{23}$

(e) $\frac{1}{25}$





$\frac{40}{20} = 2$ ← base of each rectangle

Area of 10th rectangle
= (base)(height)
= $2 f(25)$
= $2\left(\frac{1}{25}\right)$
= $\frac{2}{25}$

20. Suppose you are given the following data points for a function $f(x)$.

x	1	2	3	4
$f(x)$	4	8	13	20

If f is a linear function on each interval between the given points, find $\int_1^4 f(x)\,dx$.

Possibilities:

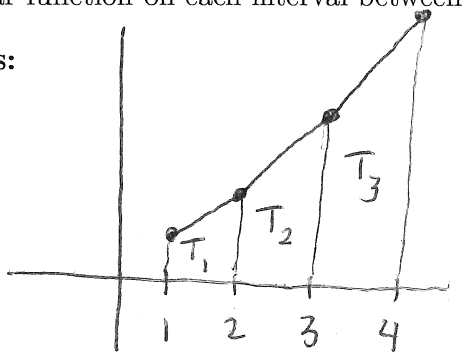
(a) 41

(b) 45

(c) 139

(d) 33

(e) 25



Area of 3 Trapezoids = $T_1 + T_2 + T_3$
= $(1)\left(\frac{4+8}{2}\right) + (1)\left(\frac{8+13}{2}\right) + (1)\left(\frac{13+20}{2}\right)$
= $\frac{12}{2} + \frac{21}{2} + \frac{33}{2}$
= $\frac{66}{2} = 33$

Some Formulas

1. **Areas:**
   (a) Triangle $A = \frac{bh}{2}$
   (b) Circle $A = \pi r^2$
   (c) Rectangle $A = lw$
   (d) Trapezoid $A = \frac{h_1 + h_2}{2} b$
2. **Volumes:**
   (a) Rectangular Solid $V = lwh$
   (b) Sphere $V = \frac{4}{3}\pi r^3$
   (c) Cylinder $V = \pi r^2 h$
   (d) Cone $V = \frac{1}{3}\pi r^2 h$

MA123 — Elem. Calculus Spring 2017
Exam 3 2017-04-13

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