| MA123 — Elem. Calculus | Fall 2018 | Name: SOLUTIONS | Sec.: ______ |
|---|---|---|---|
| Final Exam | 2018-12-10 | | |

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and twenty multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

**(a)** (b) (c) (d) (e)

You have two hours to do this exam. Please write your name and section number on this page.

**GOOD LUCK!**

3. (a) **(b)** (c) (d) (e)
4. (a) (b) (c) (d) **(e)**
5. (a) (b) (c) **(d)** (e)
6. (a) **(b)** (c) (d) (e)
7. (a) (b) **(c)** (d) (e)
8. (a) (b) (c) (d) **(e)**
9. (a) (b) (c) **(d)** (e)
10. **(a)** (b) (c) (d) (e)
11. (a) (b) (c) (d) **(e)**
12. (a) (b) **(c)** (d) (e)
13. **(a)** (b) (c) (d) (e)
14. (a) **(b)** (c) (d) (e)
15. (a) (b) (c) **(d)** (e)
16. (a) (b) **(c)** (d) (e)
17. **(a)** (b) (c) (d) (e)
18. (a) **(b)** (c) (d) (e)
19. (a) (b) **(c)** (d) (e)
20. (a) (b) (c) (d) **(e)**
21. (a) **(b)** (c) (d) (e)
22. **(a)** (b) (c) (d) (e)

**For grading use:**

| Multiple Choice | | Short Answer |
|---|---|---|
| | | |
| (number right) | (5 points each) | (out of 10 points) |

| Total | |
|---|---|
| | (maximum 110 points) |

## Fall 2018 Exam 4 Short Answer Questions

*Write answers on this page. You must show appropriate legible work to be sure you will get full credit.*

1. Find the equation of the tangent line to the graph of $f(x) = \ln(3x+10)$ at $x = 0$.

$X = 0$

$\Rightarrow y = \ln(3 \cdot 0 + 10)$

$= \ln(10)$

$\approx 2.302585$

$f'(x) = \frac{1}{3x+10} \cdot 3$

$f'(0) = \frac{1}{3 \cdot 0 + 10} \cdot 3$

$= \frac{1}{10} \cdot 3$

$= 0.3$

$y - \ln(10) = 0.3(x - 0)$

$+\ln(10) \quad +\ln(10)$

$y = 0.3x + \ln(10)$

OR $y = \frac{3}{10}x + \ln(10)$

Equation: $y =$ $0.3x + \ln(10)$

2. Evaluate $\int_0^T \left(x^{10} + \sqrt[3]{x} + 40\right)dx$. Show steps clearly and circle your final answer. You do **NOT** need to simplify your final answer.

$\int_0^T (x^{10} + \sqrt[3]{x} + 40)\,dx$

$= \int_0^T (x^{10} + x^{1/3} + 40)\,dx$

$= \frac{x^{11}}{11} + \frac{3}{4} \cdot x^{4/3} + 40x \Big|_0^T$

$= \frac{T^{11}}{11} + \frac{3T^{4/3}}{4} + 40T - (0 + 0 + 0)$

$= \frac{T^{11}}{11} + \frac{3T^{4/3}}{4} + 40T$

Name: ______________________

## Multiple Choice Questions

*Show all your work on the page where the question appears.*
*Clearly mark your answer both on the cover page on this exam*
*and in the corresponding questions that follow.*

3. Suppose you are given the following data points for a function $f(x)$.

| $x$ | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| $f(x)$ | 7 | 11 | 15 | 18 | 27 | 31 |

Use this data and a **left-endpoint** Riemann sum with five equal subdivisions to estimate the integral, $\int_0^{10} f(x)\,dx$ .

**Possibilities:**

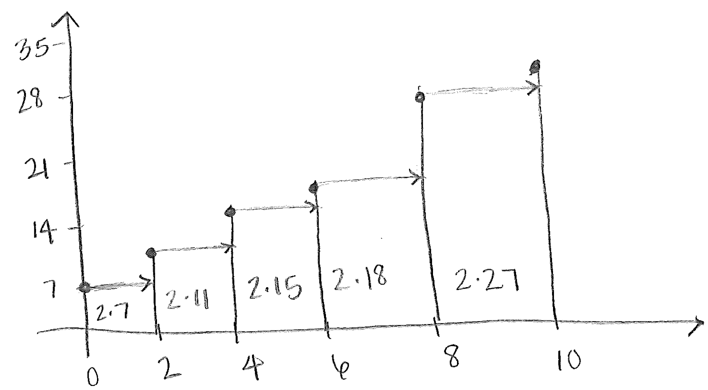
(a) 109

(b) 156

(c) 218

(d) 204

(e) 180




$14 + 22 + 30 + 36 + 54 = \boxed{156}$

4. Suppose that $\int_7^{16} f(x)\,dx = 117$. Find the average value of $f(x)$ on $[7, 16]$.

**Possibilities:**

(a) 117

(b) 14

(c) 9

(d) $\frac{117}{2}$

(e) 13

$[7, 16]$ has length $16 - 7 = 9$

So the average value is $\frac{117}{9} = \boxed{13}$

5. Assuming $x > 0$, evaluate the definite integral

$$\int_9^x \frac{13}{t^8}\, dt$$

$$= 13 \int_9^x \frac{1}{t^8}\, dt$$

$$= 13 \int_9^x t^{-8}\, dt$$

$$= 13 \left(\frac{t^{-7}}{-7}\right)\Big|_9^x$$

$$= 13\left(\frac{x^{-7}}{-7} - \frac{9^{-7}}{-7}\right)$$

$$= \boxed{-\frac{13x^{-7}}{7} + \frac{9^{-7}}{7}}$$

**Possibilities:**

(a) $26\sqrt{x} - 26\sqrt{9}$

(b) $-\frac{13}{9}(x^{-9}) + \frac{13}{9}(9^{-9})$

(c) $13\ln(|x^8|) - 13\ln(9^8)$

(d) $-\frac{13}{7}(x^{-7}) + \frac{13}{7}(9^{-7})$

(e) $\frac{13}{\frac{1}{7}x^7} - \frac{91}{4782969}$

6. Given the function $f(x) = \begin{cases} \dfrac{1}{x} & \text{if } x < 68 \\ 14x & \text{if } x \geq 68 \end{cases}$

evaluate the definite integral

$$\int_1^{78} f(x)\, dx$$

$$\int_1^{78} f(x)\,dx = \int_1^{68} f(x)\,dx + \int_{68}^{78} f(x)\,dx$$

$$= \ln(x)\Big|_1^{68} + \frac{14x^2}{2}\Big|_{68}^{78}$$

$$= \ln(x)\Big|_1^{68} + 7x^2\Big|_{68}^{78}$$

$$= \ln(68) - \ln(1) + 7(78^2) - 7(68^2)$$

$$\boxed{= \ln(68) + 10220}$$

**Possibilities:**

(a) 10658

(b) $\ln(68) + 10220$

(c) $\ln(68) + 140$

(d) $\frac{694893}{68}$

(e) 730

7. Let

$$F(x) = \int_0^x \left(t^2 + t - 42\right)\ dt$$

For which positive value of $x$ does $F'(x) = 0$?

**Possibilities:**

(a) 42

(b) $\frac{1}{2}$

$\boxed{\text{(c) } 6}$

(d) $-\frac{1}{2}$

(e) 48

$F'(x) = t^2 + t - 42$

$\Rightarrow t^2 + t - 42 = 0$

$(t+7)(t-6) = 0$

$t = -7, \boxed{t = 6 \quad C}$

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8. Use the Fundamental Theorem of Calculus to compute the derivative, $F'(x)$, of $F(x)$, if

$$F(x) = \int_1^{\boxed{8x+9}} (\ln(t))^3\ dt$$

FTOC says

If $F(x) = \int_a^{\boxed{x}} f(t)\,dt$,

then $F'(x) = f(\boxed{x})$.

So, $F'(x) = [\ln(8x+9)]^3 \cdot 8$

Chain rule

**Possibilities:**

(a) $(\ln(x))^3 \cdot (8x + 9)$

(b) $\left(\frac{1}{8x+9}\right)^3 \cdot (8)$

(c) $(\ln(x))^3 \cdot (8x + 9) \cdot (8)$

(d) $\frac{1}{4}(\ln(8x + 9))^4 \cdot (8)$

$\boxed{\text{(e) } (\ln(8x + 9))^3 \cdot (8)}$

9. Evaluate the integral

$$\int_0^T x^2 e^{9x^3+8}\, dx$$

**Possibilities:**

(a) $\frac{1}{9}e^T - \frac{1}{9}$

(b) $\frac{1}{27}e^T - \frac{1}{27}$

(c) $\frac{1}{9}e^{9T^3+8}$

(d) $\boxed{\frac{1}{27}e^{9T^3+8} - \frac{1}{27}e^8}$

(e) $9T^2e^{9T^3+8}$

Let $u = 9x^3+8$.

$du = 27x^2\,dx \Rightarrow dx = \frac{du}{27x^2}$

plug in and evaluate!

$\int_0^T x^2 e^{9x^3+8}\,dx \Rightarrow \int_{x=0}^{x=T} x^2 \cdot e^u \cdot \frac{du}{27x^2}$

$\Rightarrow \frac{1}{27}\int_{x=0}^{x=T} e^u\,du$

$\Rightarrow \frac{1}{27} e^u \Big|_{x=0}^{x=T} \Rightarrow \frac{1}{27} e^{9x^3+8} \Big|_0^T$

$\Rightarrow \frac{1}{27}\left(e^{9T^3+8} - e^8\right)$

$\Rightarrow \boxed{\frac{1}{27}e^{9T^3+8} - \frac{1}{27}e^8}$

10. A car is traveling due east. Its velocity (in miles per hour) at time t hours is given by

$$v(t) = -2.7t^2 + 16t + 60.$$

How far did the car travel during the first 7 hours of the trip?

**Possibilities:**

(a) $\boxed{\text{503.3 miles}}$

(b) 488.6 miles

(c) 69.8 miles

(d) 40.4 miles

(e) 19.6 miles

$\int_0^7 v(t)\,dt$

$= \int_0^7 -2.7t^2 + 16t + 60\,dt$

$= \frac{-2.7t^3}{3} + \frac{16t^2}{2} + \frac{60t}{1} \Big|_0^7$

$= \frac{-2.7(7)^3}{3} + \frac{16(7)^2}{2} + \frac{60(7)}{1} - 0$

$= \boxed{503.3}$

11. The graph of $y = f(x)$ shown below consists of straight lines. Evaluate the definite integral $\int_{-3}^{3} f(x)\, dx$.

**Possibilities:**

(a) 6

(b) 2.5

(c) 7.5

(d) 21.5

$\boxed{\text{(e) } 1.5}$

$$\int_{-3}^{3} f(x)\,dx$$

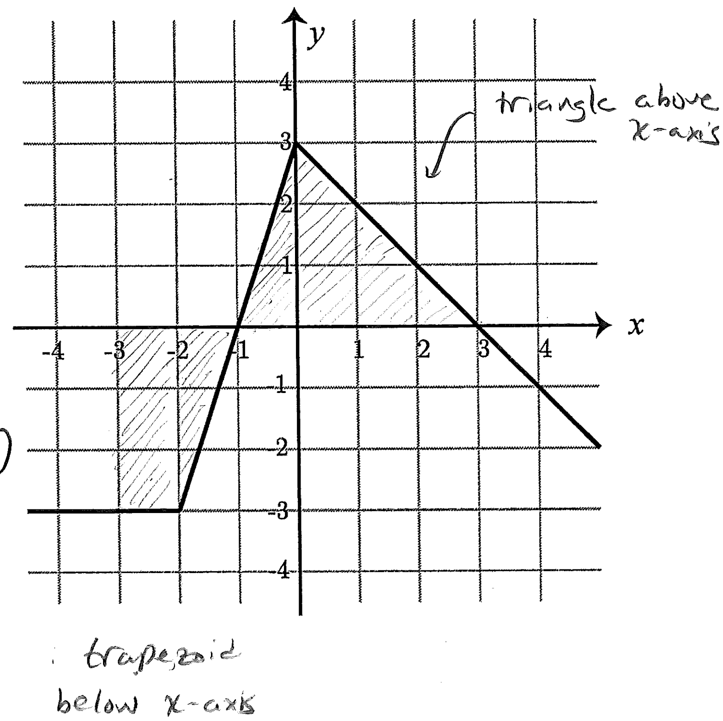
$$= \int_{-3}^{-1} f(x)\,dx + \int_{-1}^{3} f(x)\,dx$$

- trapezoid + triangle

$$= -3\left(\frac{1+2}{2}\right) + \frac{1}{2}(4)(3)$$

$$= -\frac{9}{2} + 6$$

$$= \frac{3}{2} = \boxed{1.5}$$




12. Suppose that $\int_{5}^{19} f(x)\, dx = 14$ and $\int_{13}^{19} f(x)\, dx = 27$. Find the value of $\int_{5}^{13} f(x)\, dx$.

**Possibilities:**

(a) 41

(b) $-\frac{13}{8}$

$\boxed{\text{(c) } -13}$

(d) 13

(e) $-41$

We know

$$\underbrace{\int_{5}^{13} f(x)\,dx}_{\text{want this}} + \underbrace{\int_{13}^{19} f(x)\,dx}_{\text{given}} = \underbrace{\int_{5}^{19} f(x)\,dx}_{\text{given}}$$

$$\int_{5}^{13} f(x)\,dx + 27 = 14$$

$$\qquad\qquad -27 \qquad -27$$

$$\Rightarrow \boxed{\int_{5}^{13} f(x)\,dx = -13}$$

13. Let $f(x) = x^4$. Find a value $c$ between $x = 0$ and $x = 9$, so that the average rate of change of $f(x)$ from $x = 0$ to $x = 9$ is equal to the instantaneous rate of change of $f(x)$ at $x = c$.

**Possibilities:**

(a) $\boxed{\frac{9}{\sqrt[3]{4}}}$
(b) 729
(c) $\frac{9}{4}$
(d) 2916
(e) $\frac{\sqrt[3]{4}}{9}$

AROC: $\frac{f(x_2)-f(x_1)}{x_2-x_1}$

$= \frac{9^4 - 0^4}{9-0}$

$= \frac{9^4}{9} = 729$

IROC: $f'(c)$

$f'(c) = 4c^3$

AROC = IROC

$\frac{729}{4} = \frac{4c^3}{4}$

$\sqrt[3]{\frac{729}{4}} = \sqrt[3]{c^3}$

$c = \frac{\sqrt[3]{729}}{\sqrt[3]{4}} = \boxed{\frac{9}{\sqrt[3]{4}}}$

14. Compute $\lim_{t \to 3} \frac{t^2 - 10t + 21}{t^2 + 3t - 18}$

**Possibilities:**

(a) $-\frac{5}{9}$
(b) $\boxed{-\frac{4}{9}}$
(c) $-\frac{1}{3}$
(d) $-\frac{2}{9}$
(e) The limit does not exist.

Apply limit! $\Rightarrow \frac{3^2 - 10\cdot 3 + 21}{3^2 + 3\cdot 3 - 18} = \frac{0}{0}$

Try factoring and simplifying!

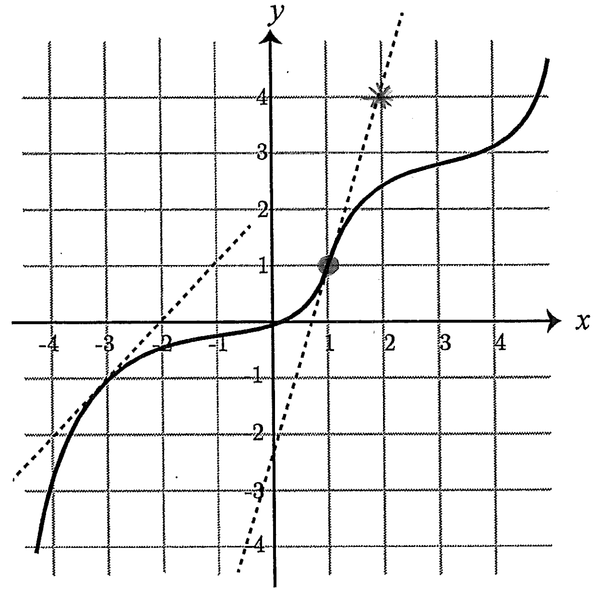
$\lim_{t \to 3} \frac{\cancel{(t-3)}(t-7)}{\cancel{(t-3)}(t+6)}$

$= \lim_{t \to 3} \frac{t-7}{t+6}$

Apply limit! $= \frac{3-7}{3+6}$

$\boxed{= \frac{-4}{9}}$

15. Determine the value of $f'(1)$ from the graph of $f(x)$ given here:




**Possibilities:**

(a) $f'(1) = -1$

(b) $f'(1) = 1$

(c) $f'(1) = -3$

(d) $f'(1) = 3$

(e) $f'(1) = 0$

$(1, f(1)) = \bullet$

Appears on dotted line so take slope between $\bullet$ and $*$.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{2-1}$

$= \frac{3}{1}$

$= 3$

$\boxed{\Rightarrow f'(1) = 3}$

16. Find the derivative, $f'(x)$, if $f(x) = (60x + 80)\ln(9x + 8)$.

**Possibilities:**

(a) $9e^{9x+8} + 60$

(b) $60\ln(9x + 8)$

(c) $(60x + 80) \cdot \frac{9}{9x+8} + 60\ln(9x + 8)$

(d) $60 \cdot \frac{9}{9x+8}$

(e) $(60x + 80) \cdot \frac{1}{9x+8} + 60\ln(9x + 8)$

product rule!

$f(x) = u \cdot v$

$f'(x) = u \cdot v' + u'v$

Here, $u = 60x + 80$, $v = \ln(9x+8)$

$u' = 60$ $\quad v' = \frac{1}{9x+8} \cdot 9$

So, $f'(x) = uv' + u'v$.

$\boxed{= (60x+80) \cdot \frac{9}{9x+8} + 60 \cdot \ln(9x+8)}$

17. The total cost (in dollars) of producing $x$ machines is $C(x) = 7000 + 60x$. The total revenue from selling $x$ machines is $R(x) = 200x - \frac{x^2}{91}$. Find the **marginal profit** function.

**Possibilities:**

(a) $140 - \frac{2}{91}x$

(b) $-7000 + 140x - \frac{x^2}{91}$

(c) $260 + 7000x - \frac{2}{91}x$

(d) $\frac{-7000}{x} + 140 - \frac{x}{91}$

(e) $\frac{7000}{x^2} - \frac{1}{91}$

"marginal" = derivative

profit = revenue − cost

$= R(x) - C(x)$

$= 200x - x^2/91 - (7000 + 60x)$

$= -x^2/91 + 140x - 7000$

marginal profit = derivative of profit

$= \boxed{-\frac{2}{91}x + 140}$

18. Suppose $H(x) = \sqrt{f(x) + g(x)}$. If $f(7) = 8$, $f'(7) = 5$, $g(7) = 28$, and $g'(7) = 9$, find $H'(7)$.

**Possibilities:**

(a) $\sqrt{14}$

(b) $\frac{7}{6}$

(c) $\frac{1}{12}$

(d) $\frac{1}{28}\sqrt{14}$

(e) 252

$H(x) = \sqrt{f(x)+g(x)} = (f(x)+g(x))^{1/2}$

$H'(x) = \frac{1}{2} \cdot (f(x)+g(x))^{-1/2} \cdot (f'(x) + g'(x))$

$= \frac{1}{2} \cdot \frac{1}{(f(x)+g(x))^{1/2}} \cdot (f'(x)+g'(x))$

$= \frac{f'(x)+g'(x)}{2\sqrt{f(x)+g(x)}}$

plug in values of functions at 7, given above.

$H'(7) = \frac{5+9}{2\sqrt{8+28}} = \frac{14}{2\sqrt{36}} = \frac{14}{12} = \boxed{\frac{7}{6}}$

19. Let $g(x) = xe^{9x}$. For $x$ in which interval(s) is $g$ concave up?

**Possibilities:**

(a) $(-\frac{1}{9}, \infty)$

(b) $(-\infty, -\frac{1}{9})$

(c) $(-\frac{2}{9}, \infty)$

(d) $(-\infty, -\frac{2}{9})$

(e) $(-\infty, \infty)$

concave up $\Rightarrow f''(x) > 0$

$g(x) = xe^{9x}$

$g'(x) = x \cdot e^{9x} \cdot 9 + 1 \cdot e^{9x}$

$= 9xe^{9x} + e^{9x}$

$= e^{9x}(9x+1)$

$g''(x) = e^{9x} \cdot 9 + e^{9x} \cdot 9 \cdot (9x+1)$

$= 9e^{9x} + (81x+9)e^{9x}$

$= (81x+18)e^{9x}$

$(81x+18)e^{9x} > 0$

$\Rightarrow 81x + 18 > 0$

$\Rightarrow 81x > -18 \Rightarrow x > -18/81 \Rightarrow \boxed{x > -2/9 \Rightarrow (-2/9, \infty)}$

20. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$. Where is the original function $f(x)$ increasing?

**Possibilities:**

(a) everywhere

(b) $(2, \infty)$

(c) nowhere

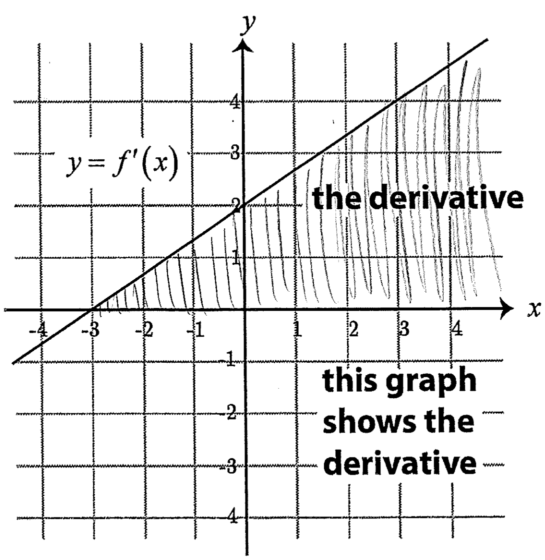
(d) $(-\infty, -3)$

(e) $(-3, \infty)$

increasing $\Rightarrow f'(x) > 0$

Anything above the x-axis means f(x) is increasing!

$\boxed{(-3, \infty)}$



21. A drug is injected into the bloodstream of a patient. The concentration of the drug in the bloodstream (in milligrams per cubic centimeter) $t$ hours after the injection is given by

$$C(t) = \frac{.17t}{t^2+6}.$$

Find the instantaneous rate of change after 1 hour.

Find $C'(1)$.

**Possibilities:**

(a) 0.024 units per hour

(b) 0.017 units per hour

(c) 0.085 units per hour

(d) 6.000 units per hour

(e) 35.294 units per hour

$$C'(t) = \frac{(t^2+6)(.17) - (.17t)(2t)}{(t^2+6)^2}$$

$$= \frac{.17t^2 + 1.02 - 0.34t^2}{(t^2+6)^2}$$

$$= \frac{-.17t^2 + 1.02}{(t^2+6)^2}$$

$$C'(1) = \frac{-.17(1) + 1.02}{(1^2+6)^2}$$

$$= \frac{.85}{49} = \boxed{0.017}$$

22. An open box is to be made out of a 12-inch by 18-inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. If we find the dimensions of the resulting box that has the largest volume, what is its **height**?

**Possibilities:**

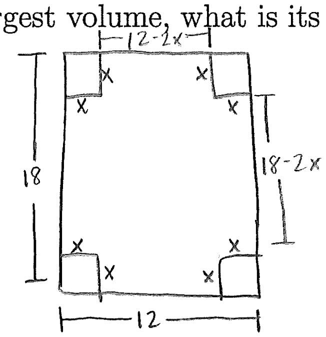
(a) 2.35 inches

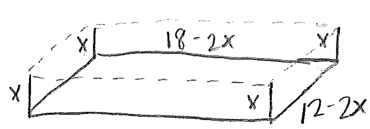
(b) 2.45 inches

(c) 2.55 inches

(d) 2.65 inches

(e) 2.75 inches




$V = lwh$

$= (12-2x)(18-2x)x$ where $0 < x < 6$

$V(x) = 216x - 60x^2 + 4x^3$

$V'(x) = 216 - 120x + 12x^2$

Find critical pts (where $V'(x) = 0$)!

$V'(x) = 216 - 120x + 12x^2 = 0$ (divide by 12)

$= 18 - 10x + x^2 = 0$

Using quadratic formula, $x = 7.65$, $x = 2.35$.

Plug in these values to find largest $V(x)$.

$V(7.65) = -68.1615$

$V(2.35) = 228.1615$

Largest volume occurs at 228.1615 when $\boxed{x = 2.35!}$

## Some Formulas

1. **Areas:**

   (a) Triangle $A = \dfrac{bh}{2}$

   (b) Circle $A = \pi r^2$

   (c) Rectangle $A = lw$

   (d) Trapezoid $A = \dfrac{h_1 + h_2}{2} b$

2. **Volumes:**

   (a) Rectangular Solid $V = lwh$

   (b) Sphere $V = \dfrac{4}{3}\pi r^3$

   (c) Cylinder $V = \pi r^2 h$

   (d) Cone $V = \dfrac{1}{3}\pi r^2 h$