MA123 — Elem. Calculus Final Exam	Spring 2016 2016-05-05	**Name:** SOLUTIONS **Sec.:** ______

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and eighteen multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name on this page, and at the top of page three.

GOOD LUCK!

3.	(a)	(b)	(c)	(d)	(e)	**12.**	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)	**13.**	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)	**14.**	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)	**15.**	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)	**16.**	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)	**17.**	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)	**18.**	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)	**19.**	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)	**20.**	(a)	(b)	(c)	(d)	(e)

For grading use:

Multiple Choice		Short Answer
(number right)	(5 points each)	(out of 10 points)

Total	
	(out of 100 points)

Spring 2016 Exam 4 Short Answer Questions

Write answers on this page. You must show appropriate legible work to be sure you will get full credit.

4 pts 1. Find the **critical numbers** (also called critical values), if any, of $f(x) = xe^{8x}$.

$$f'(x) = x \cdot e^{8x}(8) + e^{8x} \cdot (1)$$
$$= e^{8x}(8x+1)$$

$f'(x) = 0$ when $8x+1=0$

$$8x = -1$$
$$\boxed{x = -1/8}$$

6 pts 2. Evaluate $\int_0^T 2x(x^2+1)^5\,dx$. Show steps clearly. You do **NOT** need to simplify your final answer.

(use u-substitution)

let $u = x^2+1$ then $\frac{du}{dx} = 2x$

so $du = 2x\,dx$

if $x=0$ then $u = 0^2+1 = 1$

if $x=T$ then $u = T^2+1$

So $\int_0^T 2x(x^2+1)^5\,dx = \int_1^{T^2+1} u^5\,du$

$$= \frac{1}{6}u^6 \Big|_1^{T^2+1}$$

$$= \boxed{\frac{1}{6}(T^2+1)^6 - \frac{1}{6}(1)^6}$$

Name: SOLUTIONS

Multiple Choice Questions

Show all your work on the page where the question appears. Clearly mark your answer both on the cover page on this exam and in the corresponding questions that follow.

3. Find the limit as n tends to infinity. Here C is a fixed real number.

$$\lim_{n\to\infty} \frac{(Cn+1)^2}{5n^3+9n^2+4n+3}$$

$$= \lim_{n\to\infty} \frac{C^2n^2}{5n^3} = \lim_{n\to\infty} \frac{C^2}{5n}$$

$$\text{"}\frac{C^2}{5\cdot\infty}\text{"} = 0$$

Possibilities:

(a) $\frac{1}{5}C^2$

(b) $\frac{1}{21}C$

(c) $\frac{1}{125}C^2$

(d) 0

(e) ∞

4. Evaluate the limit as n tends to infinity. Note: you will have to use some of the summation formulas (see formula sheet on backpage) to simplify.

$$\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n}\frac{9k^2}{n^2}$$

$$\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n}\frac{9k^2}{n^2} = \lim_{n\to\infty} \frac{1}{n}\cdot\frac{9}{n^2}\sum_{k=1}^{n} k^2$$

$$= \lim_{n\to\infty} \frac{9}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$= \lim_{n\to\infty} \frac{18n^3}{6n^3} = \lim_{n\to\infty} 3 = 3$$

Possibilities:

(a) 3

(b) 2

(c) 1

(d) 4

(e) 5

5. Assuming $x > 0$, evaluate the definite integral

$$\int_5^x \frac{7}{t^3}\,dt$$

$$= 7\int_5^x \frac{1}{t^3}\,dt = 7\int_5^x t^{-3}\,dt$$

$$= 7\left(\frac{1}{-2}t^{-2}\right)\Bigg|_5^x$$

$$= \frac{-7}{2}x^{-2} - \frac{-7}{2}(5)^{-2}$$

$$= \frac{-7}{2}x^{-2} + \frac{7}{2}(5)^{-2}$$

Possibilities:

(a) $7\ln(|x^3|) - 7\ln(5^3)$

(b) $14\sqrt{x} - 14\sqrt{5}$

(c) $-\frac{7}{2}(x^{-2}) + \frac{7}{2}(5^{-2})$

(d) $7\sqrt{x}$

(e) $\frac{7}{\frac{1}{4}x^4} - \frac{28}{625}$

6. Find the average of $f(x) = x^2$ over [1,17].

$$\frac{1}{17-1}\int_1^{17} x^2\,dx = \frac{1}{16}\int_1^{17} x^2\,dx$$

$$= \frac{1}{16}\left(\frac{1}{3}x^3\right)\Bigg|_1^{17}$$

$$= \frac{1}{48}(17)^3 - \frac{1}{48}(1)^3$$

$$= \frac{4912}{48} \approx 102.33$$

Possibilities:

(a) 102.33

(b) 18.00

(c) 144.50

(d) 145.00

(e) 1637.33

7. Find the value of x at which

$$F(x) = \int_3^x (|t| + 4)\ dt$$

takes its minimum value on the interval $[8, 900]$.

Possibilities:

(a) 900

(b) 8

(c) 3

(d) 12

(e) 408536.0

$F'(x) = \frac{d}{dx}\int_3^x (|t|+4)\ dt$

$= |x|+4$

$F'(x) \geq 0$ for all x

this $F(x)$ is always increasing

$\Rightarrow$ minimum value occurs at left endpoint of interval

8. Evaluate the integral

$$\int_0^x (4t+8)^{15}\ dt$$

Possibilities:

(a) $\frac{1}{16}(4x+8)^{16} - \frac{8^{16}}{16}$

(b) $\frac{1}{4(16)}(4x+8)^{16} - \frac{8^{16}}{4(16)}$

(c) $16(4x+8)^{16} - 15 \cdot 8^{16}$

(d) $\frac{1}{15}(4x+8)^{15} - \frac{8^{15}}{15}$

(e) $\frac{1}{16}x^{16} - \frac{8^{16}}{16}$

Let $u = 4t+8$

$\frac{du}{dt} = 4$ $\quad du = 4\,dt$

$\frac{1}{4}du = dt$

if $t=0$, $u = 4(0)+8 = 8$

if $t=x$ $\quad u = 4x+8$

$\int_0^x (4t+8)^{15}\ dt = \int_8^{4x+8} \frac{1}{4}u^{15}\ du$

$= \frac{1}{4}\int_8^{4x+8} u^{15}\ du = \frac{1}{4}\left(\frac{1}{16}u^{16}\right)\Big|_8^{4x+8}$

$= \frac{1}{4}\left[\frac{1}{16}(4x+8)^{16} - \frac{1}{16}(8)^{16}\right]$

9. A car is traveling due east. Its velocity (in miles per hour) at time t hours is given by $v(t) = -2.4t^2 + 14t + 60$. How far did the car travel during the first 5 hours of the trip?

Possibilities:

(a) 10.0 miles

(b) 75.0 miles

(c) 375.0 miles

(d) 350.0 miles

(e) 70.0 miles

$$\int_0^5 (-2.4t^2 + 14t + 60)\,dt$$

$$= \left(\frac{-2.4}{3}t^3 + \frac{14}{2}t^2 + 60t\right)\Big|_0^5$$

$$= -.8(5)^3 + 7(5)^2 + 60(5) - [0]$$

$= 375$ miles

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10. Compute $\lim_{t\to 3} \dfrac{t^2 + 4t - 21}{t^2 + 5t - 24}$

Possibilities:

(a) $\frac{8}{11}$

(b) $\frac{9}{11}$

(c) $\frac{10}{11}$

(d) 1

(e) The limit does not exist.

plug in 3 $\rightarrow$ "$\frac{0}{0}$"

$$\lim_{t\to 3} \frac{t^2+4t-21}{t^2+5t-24} = \lim_{t\to 3} \frac{(t+7)\cancel{(t-3)}}{(t+8)\cancel{(t-3)}}$$

$$= \lim_{t\to 3} \frac{t+7}{t+8} = \frac{3+7}{3+8} = \frac{10}{11}$$

11. Let $f(x) = 2x^2 + 3x + 7$. Find a value c between $x = 4$ and $x = 8$, so that the average rate of change of $f(x)$ from $x = 4$ to $x = 8$ is equal to the instantaneous rate of change of $f(x)$ at $x = c$.

Possibilities:

(a) 5
(b) 6
(c) 7
(d) 8
(e) 9

ARoC = IRoC

$$\frac{f(8)-f(4)}{8-4} = f'(c)$$

$$\frac{\left[2(8)^2+3(8)+7\right]-\left[2(4)^2+3(4)+7\right]}{8-4} = 4c+3$$

$$\frac{108}{4} = 4c+3$$

$$27 = 4c+3$$

$$24 = 4c$$

$$6 = c$$

12. How many years will it take an investment to triple in value if the interest rate is 9% compounded continuously?

Possibilities:

(a) 12.21 years
(b) 13.73 years
(c) 15.69 years
(d) 18.31 years
(e) 21.97 years

$$P(t) = P_0 e^{rt}$$

$$3P_0 = P_0 e^{.09t}$$

$$3 = e^{.09t}$$

$$\ln 3 = \ln e^{.09t}$$

$$\ln 3 = .09t$$

$$\frac{\ln 3}{.09} = t$$

$$t \approx 12.2068$$

13. The tangent line to the graph of f at $x = 4$ has equation $y = 8(x - 4) + 3$. Find $f(4)$ and $f'(4)$.

Possibilities:

(a) $f(8) = 3, \quad f'(8) = 4$

(b) $f(4) = 8, \quad f'(4) = 3$

(c) $f(3) = 8, \quad f'(3) = 4$

(d) $f(3) = 4, \quad f'(3) = 8$

(e) $f(4) = 3, \quad f'(4) = 8$

$y = 8(x-4)+3$

$y - 3 = \underline{\underline{8}}(x-4)$

$f'(4) = 8$

line contains point $(4,3)$

thus $f(4) = 3$

14. The graph of $y = f(x)$ is shown below. The function is continuous, except at $x =$

Possibilities:

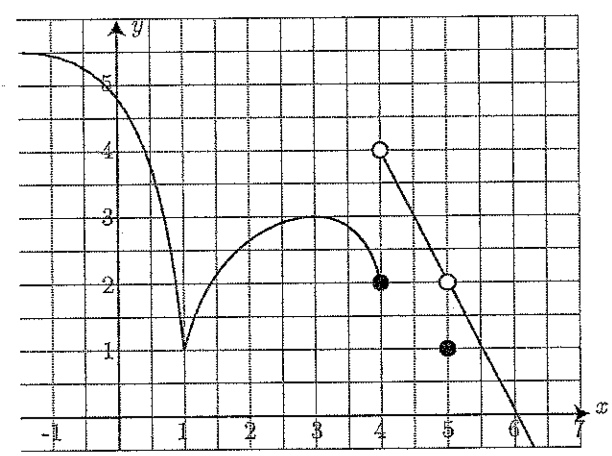
(a) x=1, x=3, x=4, and x=5

(b) x=1, x=4, and x=5

(c) x=4 and x=5

(d) x=4 only

(e) x=1 and x=3



look for holes, skips, jumps in the graph

jump at $x=4$

hole at $x=5$

15. If $f(x) = 6x^4 + 2x^2 + 3x$ then find the second derivative $f''(x)$:

Possibilities:

(a) $96x^4 + 8x^2$

(b) $24x^3 + 4x + 3$

(c) $72x^2 + 16$

(d) $24x^3 + 36x^2 + 28x + 11$

(e) $72x^2 + 4$

$f'(x) = 24x^3 + 4x + 3$

$f''(x) = 72x^2 + 4$

16. Find the derivative, $f'(x)$, if $f(x) = (2 + 6x)\ln(7 + 3x)$.

Possibilities:

(a) $(6)\ln(7 + 3x) + \dfrac{2 + 6x}{x}$

(b) $\dfrac{6}{7 + 3x}$

(c) $6 + \dfrac{3}{7 + 3x}$

(d) $(6)\ln(7 + 3x) + \dfrac{6 + 18x}{7 + 3x}$

(e) $\dfrac{9}{7 + 3x}$

product rule

$f'(x) = (2+6x)' \cdot \ln(7+3x) + (2+6x)[\ln(7+3x)]'$

$= 6 \cdot \ln(7+3x) + (2+6x)\dfrac{1}{7+3x}(3)$

$= 6\ln(7+3x) + \dfrac{3(2+6x)}{7+3x}$

17. Suppose $F(x) = (g(x))^3 + 9$. If $g(2) = 7$, $g'(2) = 13$, and $g''(2) = 5$, then find $F'(2)$.

Possibilities:

(a) $(3)(7^2) + 9$

(b) $7^3 + 9$

(c) 5

(d) $13^3 + 9$

(e) $(3)(7^2)(13)$

$$F'(x) = 3(g(x))^2 (g'(x))$$

$$F'(2) = 3(g(2))^2 g'(2)$$

$$= 3(7)^2(13)$$

$$= 1911$$

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18. Suppose the derivative of $g(t)$ is $g'(t) = -12(t-4)(t-8)$. For t in which interval(s) is g concave up?

Possibilities:

(a) $(-\infty, 6)$

(b) $(6, \infty)$

(c) $(-\infty, 4) \cup (8, \infty)$

(d) $(4, 6) \cup (8, 12)$

(e) $(4, 8)$

use product rule

$$g''(t) = -12[(1)(t-8) + (t-4)(1)]$$

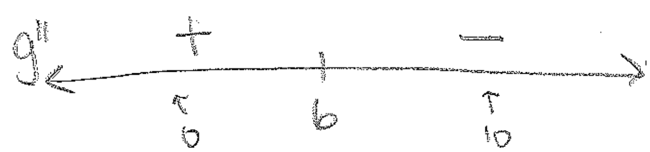
$$= -12[2t - 12]$$

$$= -24t + 144$$

$g''(t) = 0$ when $-24t + 144 = 0$

$144 = 24t$

$6 = t$



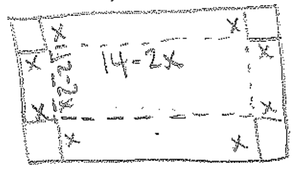
$g''(0) = -24(0) + 144 = 144$

$g''(10) = -24(10) + 144 = -96$

$g(t)$ is concave up when $g''(t)$ is "+"

$(-\infty, 6)$

19. An open box is to be made out of a 12-inch by 14-inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. If we find the dimensions of the resulting box that has the largest volume, what is its height?



$V = (14-2x)(12-2x)(x)$

$V = (168 - 52x + 4x^2)(x)$

$V = 4x^3 - 52x^2 + 168x$

Possibilities:

(a) 1.85 inches

(b) 1.95 inches

(c) 2.05 inches

(d) 2.15 inches

(e) 2.25 inches

$14-2x \geq 0$, $x \leq 7$ $\quad$ $12-2x \geq 0$, $x \leq 6$ $\quad$ $x \geq 0$ $\quad$ so $x \in [0,6]$

"find max volume on interval $[0,6]$"

$V' = 12x^2 - 104x + 168$

$= 4(3x^2 - 26x + 42)$

$V' = 0$ when

$x = \frac{26 \pm \sqrt{26^2 - 4(3)(42)}}{2(3)} = \frac{26 \pm \sqrt{172}}{6}$

* $\frac{26+\sqrt{172}}{6}$ not in interval

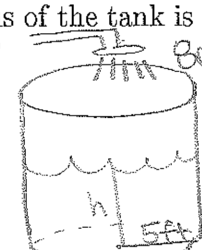
Check $V(0) = (14)(12)(0) = 0$

$V(6) = (2)(0)(6) = 0$

$V\left(\frac{26-\sqrt{172}}{6}\right) \approx 160.584 \leftarrow$ max

max occurs at $x = \frac{26-\sqrt{172}}{6} \approx 2.147$

20. A cylindrical water tank with its circular base parallel to the ground is being filled at the rate of 80 cubic feet per minute. The radius of the tank is 5 feet. How fast is the level of the water in the tank rising when the tank is half full?



Possibilities:

(a) 12566.37 feet per minute

(b) 0.51 feet per minute

(c) 1.02 feet per minute

(d) 6283.19 feet per minute

(e) 2513.27 feet per minute

volume of cylinder $V = \pi r^2 h$

$V = \pi (5)^2 h = 25\pi h$

take derivative with respect to time

$\frac{dV}{dt} = 25\pi \frac{dh}{dt}$

$80 = 25\pi \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{80}{25\pi} \approx 1.01859$ ft/min

Some Formulas

1. Summation formulas:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

2. Areas:

(a) Triangle $A = \frac{bh}{2}$

(b) Circle $A = \pi r^2$

(c) Rectangle $A = lw$

(d) Trapezoid $A = \frac{h_1 + h_2}{2} b$

3. Volumes:

(a) Rectangular Solid $V = lwh$

(b) Sphere $V = \frac{4}{3}\pi r^3$

(c) Cylinder $V = \pi r^2 h$

(d) Cone $V = \frac{1}{3}\pi r^2 h$

4. Distance:

(a) Distance between (x_1, y_1) and (x_2, y_2)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$