MA123 — Elem. Calculus Spring 2017
Final Exam 2017-05-03

Name: Solutions Sec.: _____

Do not remove this answer page — you will turn in the entire exam. No books or notes may be used. You may use an ACT-approved calculator during the exam, but NO calculator with a Computer Algebra System (CAS), networking, or camera is permitted. Absolutely no cell phone use during the exam is allowed.

The exam consists of two short answer questions and twenty multiple choice questions. Answer the short answer questions on the back of this page, and record your answers to the multiple choice questions on this page. For each multiple choice question, you will need to fill in the circle corresponding to the correct answer. It is your responsibility to make it CLEAR which response has been chosen. For example, if (a) is correct, you must write

(a) (b) (c) (d) (e)

You have two hours to do this exam. Please write your name and section number on this page.

GOOD LUCK!

3.	a	**b**	c	d	e	13.	a	**c**	d	e
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12.	a	b	**c**	d	e	22.	a	b	c	**d**	e

For grading use:

Multiple Choice		Short Answer
(number right)	(5 points each)	(out of 10 points)

Total	
	(max 110 points)

Solutions

Spring 2017 Exam 4 Short Answer Questions

Write answers on this page. You must show appropriate legible work to be sure you will get full credit.

1. Find the equation of the tangent line to the graph of $f(x)=(5x+2)^4$ at $x=0$.

Need slope and point

Slope $= f'(0)$

$f'(x) = 4(5x+2)^3 \cdot 5$

$f'(0) = 4(2)^3 \cdot 5$

$= 20(8) = 160$

point: $x = 0$

$y = f(0) = (2)^4 = 16$

$(x, y) = (0, 16)$

Write equation:

$y - 16 = 160(x - 0)$

OR in $y = mx + b$ form

$y = 160x + 16$

Equation: $y =$ $160x + 16$

2. Evaluate $\int_1^T \left(x^3 + \frac{1}{x^{12}}\right) dx$. Show steps clearly and circle your final answer. You do **NOT** need to simplify your final answer.

$$\int_1^T x^3 + x^{-12}\, dx$$

$$= \frac{x^4}{4} - \frac{x^{-11}}{11}\Bigg|_1^T$$

$$= \left(\frac{T^4}{4} - \frac{T^{-11}}{11}\right) - \left(\frac{1}{4} - \frac{1}{11}\right)$$

Name: Solutions

Multiple Choice Questions

Show all your work on the page where the question appears.
Clearly mark your answer both on the cover page on this exam
and in the corresponding questions that follow.

3. Suppose you are given the following data points for a function $f(x)$.

x	0	2	4	6	8	10
$f(x)$	7	8	13	20	24	32

Use this data and a **left-endpoint** Riemann sum with five equal subdivisions to estimate the integral, $\int_0^{10} f(x)\,dx$.

Possibilities:

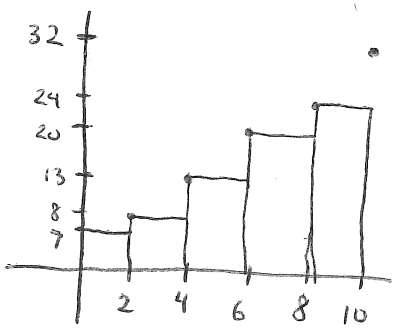
(a) 104

(b) 144

(c) 169

(d) 208

(e) 194



Rectangle #	base	height	Area
1	2	7	14
2	2	8	16
3	2	13	26
4	2	20	40
5	2	24	48
Total			144

4. Suppose that the average value of $f(x)$ on $[6, 10]$ is 68. Find the value of $\int_6^{10} f(x)\,dx$.

Possibilities:

(a) 302

(b) 544

(c) 272

(d) 2176

(e) 136

Average value of f on $[6,10]$ $= \dfrac{\int_6^{10} f(x)\,dx}{10-6} = 68$

$\dfrac{\int_6^{10} f(x)\,dx}{4} = 68$

$\int_6^{10} f(x)\,dx = 68(4) = 272$

5. Evaluate the definite integral

$$\int_2^x 12\sqrt{t}\ dt$$

Possibilities:

(a) $12\sqrt{x}$

(b) $12x^{\frac{3}{2}} - 12 \cdot 2^{\frac{3}{2}}$

(c) $24\sqrt{x} - 24\sqrt{2}$

(d) $\frac{12}{\sqrt{x}} - \frac{12}{\sqrt{2}}$

(e) $8x^{\frac{3}{2}} - 8 \cdot 2^{\frac{3}{2}}$

$= \int_2^x 12t^{\frac{1}{2}}\,dt$

$= 12\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)\Big|_2^x$

$= 12\left(\frac{2}{3}\right)t^{\frac{3}{2}}\Big|_2^x$

$= 8t^{\frac{3}{2}}\Big|_2^x$

$= 8x^{\frac{3}{2}} - 8(2)^{\frac{3}{2}}$

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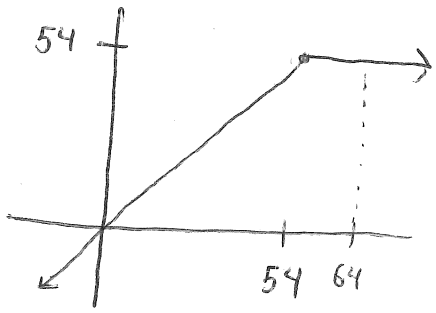
6. Given the function $f(x) = \begin{cases} x & \text{if } x < 54 \\ 54 & \text{if } x \geq 54 \end{cases}$

evaluate the definite integral

$$\int_0^{64} f(x)\ dx$$



Possibilities:

(a) 1995

(b) 1996

(c) 1997

(d) 1998

(e) 1999

$= \int_0^{54} f(x)\,dx + \int_{54}^{64} f(x)\,dx$

$= \int_0^{54} x\,dx + \int_{54}^{64} 54\,dx$

$= \frac{x^2}{2}\Big|_0^{54} + 54x\Big|_{54}^{64}$

$= \left(\frac{(54)^2}{2} - 0\right) + 54(64) - 54(54) = 1998$

---

7. Let

$$F(x) = \int_0^x (t^2 - 7t)\ dt$$

For which positive value of x does $F'(x) = 0$?

Possibilities:

(a) $\frac{7}{2}$

(b) $\frac{21}{2}$

(c) 7

(d) 0

(e) $\frac{665}{6}$

Use Fund. Thm. of Calculus

$\frac{d}{dx} F(x) = \frac{d}{dx}\left(\int_0^x t^2 - 7t\ dt\right)$

$F'(x) = x^2 - 7x$

$0 = x^2 - 7x$

$0 = x(x-7)$

$x = 0, 7$

$x = 7$ is the only positive value

8. Suppose a rock is dropped from a martian cliff. After t seconds, its speed in feet per second is $v(t) = \frac{61}{5}t$, at least until it lands. If the rock lands after 8 seconds, how high (in feet) is the cliff?

Possibilities:

(a) $\frac{61}{40}$ feet

(b) 8 feet

(c) 4 feet

(d) $\frac{488}{5}$ feet

(e) $\frac{1952}{5}$ feet

displacement $(s(t))$ from $t=0$ to $t=8$ is $\int_0^8 v(t)dt$.

This tells us how far the rock fell and thus the height of the cliff.

$\text{Height} = \int_0^8 \frac{61}{5} t\ dt$

$= \frac{61}{5}\left(\frac{t^2}{2}\right)\Big|_0^8$

$= \frac{61}{10}(8)^2 - 0$

$= \boxed{\frac{1952}{5}\ ft}$

9. Evaluate the integral

$$\int_0^T 6e^{6x+2}\,dx$$

Possibilities:

(a) $6e^{6T+2} - 6e^2$

(b) $6e^{6T+2}$

(c) $6e^T - 6$

(d) $e^{6T+2} - e^2$

(e) $\frac{6}{3}e^{6T+3}$

Use u-substitution

Let $u = 6x+2 \Rightarrow \frac{du}{dx} = 6$

$du = 6\,dx$

If $x=0$, then $u = 6(0)+2 = 2$

If $x=T$, then $u = 6T+2$

Thus, $\int_0^T 6e^{6x+2}\,dx = \int_2^{6T+2} e^u\,du = e^u\Big|_2^{6T+2} = e^{6T+2} - e^2$

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10. Suppose that $\int_6^{23} f(x)\,dx = 8$. Find the value of $\int_6^{23} (3f(x)+2)\,dx$.

Possibilities:

(a) 58

(b) 41

(c) 70

(d) 26

(e) 30

$= 3\int_6^{23} f(x)\,dx + \int_6^{23} 2\,dx$

$= 3(8) + 2x\Big|_6^{23}$

$= 24 + 2(23) - 2(6)$

$= 24 + 34$

$= 58$

11. The graph of $y = f(x)$ shown below includes a semicircle and a straight line. Evaluate the definite integral $\int_{-4}^{4} f(x)\,dx$. Use $\pi = 3.14$.

Signed Area

Possibilities:

(a) -14.28

(b) 14.28

(c) -4.56

(d) 1.72

(e) $-.28$

$= \text{Area of triangle} - \text{Area of Semicircle}$

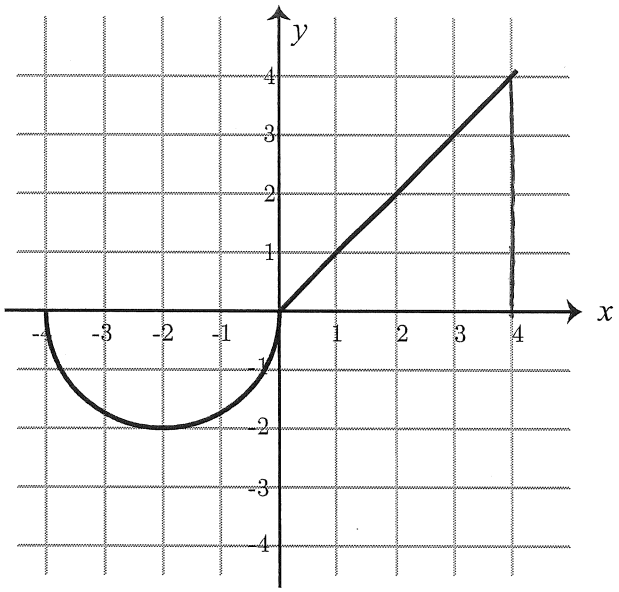
$= \frac{1}{2}(4)(4) - \frac{1}{2}\pi(2)^2$

$= 8 - 2\pi$

$\approx 8 - 2(3.14)$

$= 8 - 6.28$

$= 1.72$



12. Let $f(x) = x^3$. Find a value c between $x = 0$ and $x = 9$, so that the average rate of change of $f(x)$ from $x = 0$ to $x = 9$ is equal to the instantaneous rate of change of $f(x)$ at $x = c$.

Possibilities:

(a) 243

(b) 7

(c) $\frac{9}{\sqrt{3}}$

(d) $\frac{9}{\sqrt{5}}$

(e) $\frac{\sqrt{3}}{9}$

$AROC = \frac{f(9) - f(0)}{9 - 0} = \frac{(9)^3 - (0)^3}{9} = 9^2 = 81$

Instantaneous rate of change of f at $x = c$ $= f'(c)$

$f'(x) = 3x^2$

$f'(c) = 3c^2$

Set $3c^2 = 81$

$c^2 = 27$

$c = \pm\sqrt{27} = \pm 3\sqrt{3} \Rightarrow c = +3\sqrt{3} = \frac{9}{\sqrt{3}}$

$\frac{3\sqrt{3}}{1}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{3(3)}{\sqrt{3}} = \frac{9}{\sqrt{3}}$

We want a value between 0 and 9, so we want c to be positive

13. Compute $\lim_{t\to 1} \dfrac{t^2+8t-9}{t^2-8t+7}$

Possibilities:

(a) $-\frac{2}{3}$

(b) $-\frac{5}{3}$

(c) 0

(d) 1

(e) The limit does not exist.

Plug in $t=1$:

$\dfrac{1+8-9}{1-8+7} = \dfrac{0}{0}$

Now try to simplify first

$$\lim_{t\to 1} \frac{(t+9)(t-1)}{(t-1)(t-7)}$$

$$= \lim_{t\to 1} \frac{t+9}{t-7}$$

Plug in $t=1$: $\dfrac{1+9}{1-7} = \dfrac{10}{-6} = -\dfrac{5}{3}$

---

14. Find the limit

$$\lim_{n\to\infty} \frac{(2n+3)^2}{17n^2+13} = \lim_{n\to\infty} \frac{4n^2+12n+9}{17n^2+13}$$

Possibilities:

(a) The limit does not exist or approaches infinity

(b) $\frac{9}{13}$

(c) $\frac{2}{17}$

(d) $\frac{4}{13}$

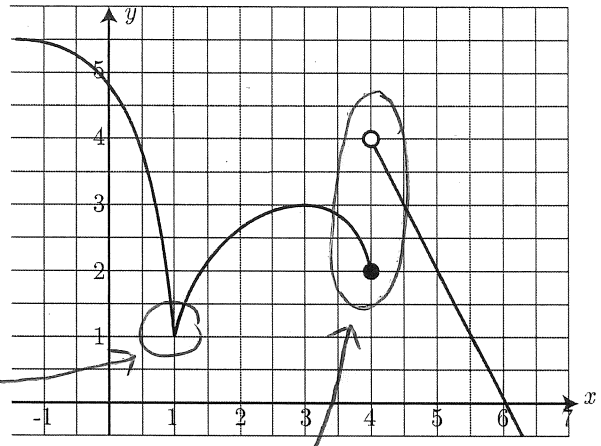
(e) $\frac{4}{17}$

We take the highest power terms for limits at $\pm\infty$

$$= \lim_{n\to\infty} \frac{4n^2}{17n^2}$$

$$= \lim_{n\to\infty} \frac{4}{17} = \frac{4}{17}$$

15. The graph of $y = f(x)$ is shown below. The function is **continuous**, except at $x =$



Possibilities:

(a) x=1 only

(b) x=1 and x=4

(c) x=4 only

(d) x=1, x=3, x=4, and x=6

(e) x=1, x=3, and x=4

f is continuous at $x=1$, but it is not differentiable there

jump at $x=4$ (f is not continuous)

16. Find the derivative, $f'(x)$, if $f(x) = (20x + 50)\ln(6x + 2)$.

Possibilities:

(a) $(20x + 50) \cdot \frac{1}{6x+2} + 20\ln(6x + 2)$

(b) $(20x + 50) \cdot \frac{6}{6x+2} + 20\ln(6x + 2)$

(c) $6e^{6x+2} + 20$

(d) $20 \cdot \frac{6}{6x+2}$

(e) $20\ln(6x + 2)$

Product Rule first

$f'(x) = (20x+50)(\ln(6x+2))' + \ln(6x+2)(20x+50)'$

↑ need Chain Rule here

$f'(x) = (20x+50)\frac{1}{6x+2} \cdot 6 + \ln(6x+2)(20)$

$= (20x+50) \cdot \frac{6}{6x+2} + 20\ln(6x+2)$

17. If $f(x) = x^7 + 6x^5 + 2x^4 + 3x^2 + 7$ then find the second derivative $f''(x)$:

Possibilities:

(a) $42x^5 + 120x^3 + 24x^2 + 6$

(b) $7x^6 + 21x^5 + 65x^4 + 103x^3 + 93x^2 + 51x + 12$

(c) $42x^5 + 190x^3 + 24x^2 + 74x + 10$

(d) $49x^7 + 150x^5 + 32x^4 + 12x^2$

(e) $7x^6 + 30x^4 + 8x^3 + 6x$

$f'(x) = 7x^6 + 30x^4 + 8x^3 + 6x$

$f''(x) = 42x^5 + 120x^3 + 24x^2 + 6$

---

18. Suppose $g(8) = 7$ and $g'(8) = 4$. Find $F'(8)$ if

$$F(x) = \frac{x^2+1}{g(x)}$$

Possibilities:

(a) $-\frac{9}{4}$

(b) 4

(c) $-\frac{144}{7}$

(d) $-\frac{148}{49}$

(e) $\frac{144}{49}$

Quotient Rule

$$F'(x) = \frac{g(x)(x^2+1)' - (x^2+1)g'(x)}{(g(x))^2}$$

$$F'(x) = \frac{g(x)(2x) - (x^2+1)g'(x)}{(g(x))^2}$$

$$F'(8) = \frac{g(8)(16) - (65)g'(8)}{(g(8))^2}$$

$$= \frac{(7)(16) - (65)(4)}{(7)^2} = -\frac{148}{49}$$

---

19. Suppose the derivative of $g(t)$ is $g'(t) = 11t^2 - 88t + 132$. For t in which interval(s) is g concave up?

Possibilities:

(a) $(-\infty, 2) \cup (6, \infty)$

(b) $(2, 6)$

(c) $(4, \infty)$

(d) $(-\infty, 4)$

(e) $(2, 4) \cup (6, 11)$

$g''(t) = 22t - 88$

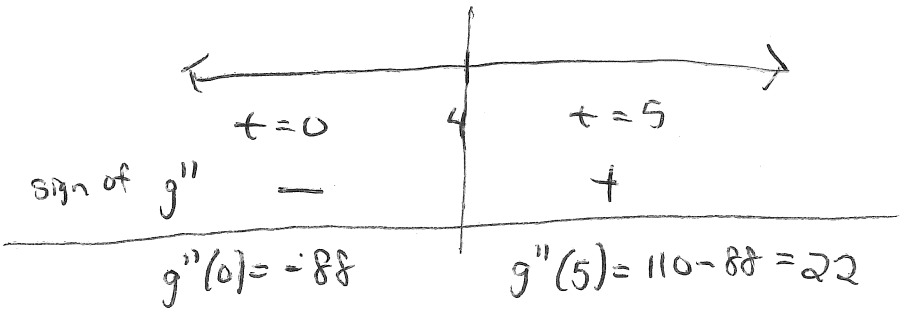
Set $g''(t) = 0$

$0 = 22t - 88$

$88 = 22t$

$4 = t$

↑ when $g''(t) > 0$



g is concave up on $(4, \infty)$ since $g''(x) > 0$

20. The following is the graph of the derivative, $f'(x)$, of the function $f(x)$. Where is the original function $f(x)$ increasing?

Possibilities:

(a) $(-1, \infty)$

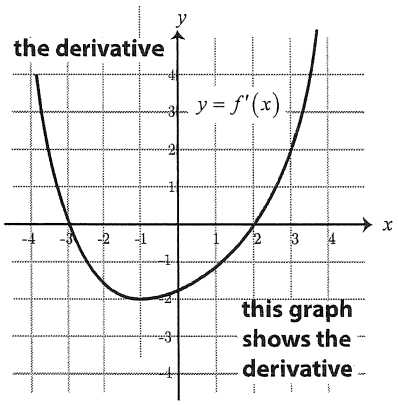
(b) $(-3, 2)$

(c) $(-2, \infty)$

(d) $(-\infty, -1)$

(e) $(-\infty, -3)$ and $(2, \infty)$

$f(x)$ is increasing when $f'(x) > 0$. This happens on $(-\infty, -3) \cup (2, \infty)$



21. Boyle's Law states that when a sample gas is compressed at a constant temperature, the pressure P and volume V satisfy the equation $PV = c$, where c is a constant. Suppose that at a certain instant the volume is 46 cubic centimeters, the pressure is 5 kPa, and the pressure is increasing at a rate of 4 kPa/min. At what rate is the volume decreasing at this instant?

Possibilities:

(a) $\frac{183}{5}$ cubic centimeters per minute

(b) $\frac{184}{5}$ cubic centimeters per minute

(c) 37 cubic centimeters per minute

(d) $\frac{186}{5}$ cubic centimeters per minute

(e) $\frac{187}{5}$ cubic centimeters per minute

$PV = C$

Take $\frac{d}{dt}$ of both sides and use Product Rule

$P\frac{dV}{dt} + V\frac{dP}{dt} = 0$ (Note $\frac{d}{dt}(c) = 0$ since c is a constant)

$5\frac{dV}{dt} + 46(4) = 0$

$5\frac{dV}{dt} = -184$

$\frac{dV}{dt} = -\frac{184}{5} \Rightarrow$ volume is decreasing at a rate of $\frac{184}{5}\ \frac{cm^3}{min}$

---

22. A landscape architect wishes to enclose a rectangular garden on one side by a brick wall costing \$50 per foot, and on the other three sides by a metal fence costing \$10 per foot. If the area of the garden is 100 square feet, find the lowest possible cost to enclose the garden.

Possibilities:

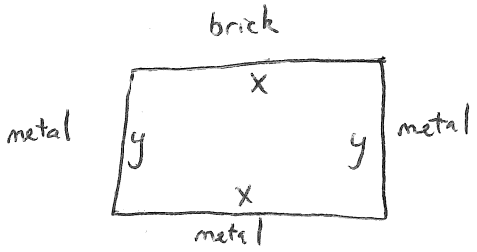
(a) \$693.32

(b) \$693.82

(c) \$692.32

(d) \$692.82

(e) \$694.32



$A = 100 = xy$

Minimize Cost (C)

$C = 50x + 10x + 10y + 10y$

$C = 60x + 20y$

Solve for one of the variables in $100 = xy$ and substitute into cost equation

$y = \frac{100}{x}$

$C = 60x + 20\left(\frac{100}{x}\right)$

$C = 60x + \frac{2000}{x}$

$C = 60x + 2000x^{-1}$

Interval for x: $(0, \infty)$

$C' = 60 - 2000x^{-2}$

$C' = 60 - \frac{2000}{x^2}$

Continued on next page

Some Formulas

1. **Areas:**
   (a) Triangle $A = \frac{bh}{2}$
   (b) Circle $A = \pi r^2$
   (c) Rectangle $A = lw$
   (d) Trapezoid $A = \frac{h_1 + h_2}{2} b$

2. **Volumes:**
   (a) Rectangular Solid $V = lwh$
   (b) Sphere $V = \frac{4}{3}\pi r^3$
   (c) Cylinder $V = \pi r^2 h$
   (d) Cone $V = \frac{1}{3}\pi r^2 h$

#22 continued

Find critical points

→ C' is undefined when $x=0$ but $x=0$ is not in our interval $(0,\infty)$

→ Set $C' = 0$

$$0 = 60 - \frac{2000}{x^2}$$

$$\frac{2000}{x^2} = 60$$

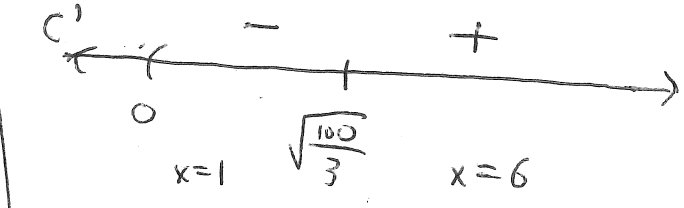
$$\frac{2000}{60} = x^2$$

$$\frac{100}{3} = x^2$$

$\Rightarrow x = \pm\sqrt{\frac{100}{3}} \approx \pm 5.77$

We take the positive value since our interval for x is $(0,\infty)$.

Test sign of C'



$C'(1) = 60 - 2000 = -1940$

$C'(6) = 60 - \frac{2000}{36} = \frac{40}{9}$

Min at $x = \sqrt{\frac{100}{3}}$ since C' changes from − to +

$$C = 60x + \frac{2000}{x}$$

Plug in $x = \sqrt{\frac{100}{3}}$ to get

$$C = 60\sqrt{\frac{100}{3}} + \frac{2000}{\sqrt{\frac{100}{3}}} \approx \$692.82$$

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