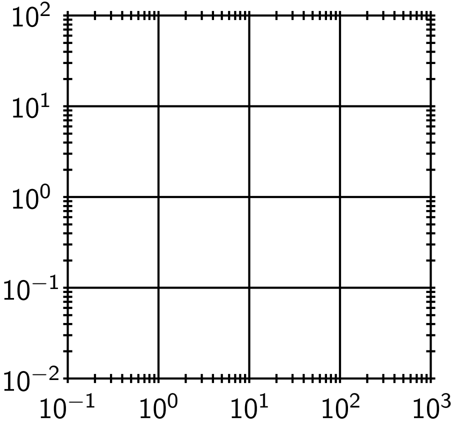
Double-log (or Log-Log) Plots

- If we use logarithmic scales on both the horizontal and vertical axes, the resulting graph is called a log-log plot.



Lines in Double-Log Plots

- A log-log plot is used when we suspect that a power function might be a good model for our data.
- Recall that power functions are frequently found in "scaling relations" between biological variables (e.g., organ sizes). Finding such relationships is the objective of **allometry**.
- If we start with a **power function** $y = Cx^p$ and take logarithms of both sides, we get

$$\log y = \log(Cx^p) = \log C + \log x^p$$

$$\log y = \log C + p \log x$$

Let $Y = \log y$, $A = \log C$, and $X = \log x$. Then the latter equation becomes

$$Y = A + pX$$

We recognize that Y is a linear function of X, so the points $(\log x, \log y)$ lie on a straight line.

Example 7: (Problem # 58, Section 1.3, p. 53)

When $\log y$ is graphed as a function of $\log x$, a straight line results. Graph the straight line given by the following two points

$$(x_1, y_1) = (2, 5) \qquad (x_2, y_2) = (5, 2)$$

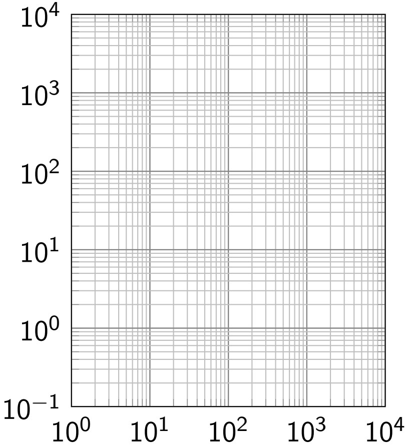
on a log-log plot. determine the functional relationship between x and y. (**Note:** The original x-y coordinates are given.)

Example 8: (Exam 1, Fall 13, # 4)

There are several possible functional relationships between height and diameter of a tree. One particularly simple model is given by

$$H = AD^{3/4}$$

where A is a constant that depends on the species of tree, H is the height, and D is the diameter. If $A = 50$ plot this relationship in the double log plot below.



Is your graph a straight line? If so, what is its slope?

Example 9: (Problem # 74, Section 1.3, p. 54)

The following table is based on a functional relationship between x and y that is either an exponential or a power function:

x	y
0.5	7.81
1	3.4
1.5	2.09
2	1.48
2.5	1.13

Use an appropriate logarithmic transformation and a graph to decide whether the table comes from a power function or an exponential function, and find the functional relationship between x and y.

Example 10 (Forgetting):

Ebbinghaus's Law of Forgetting states that if a task is learned at a performance level P_0, then after a time interval t the performance level P satisfies

$$\log P = \log P_0 - c\log(t+1),$$

where c is a constant that depends on the type of task and t is measured in months.

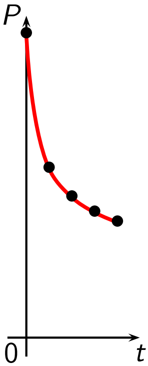
(a) Solve the equation for P.

(b) Use Ebbinghaus's Law of Forgetting to estimate a student's score on a biology test two years after he got a score of 80 on a test covering the same material. Assume $c = 0.3$.

Comment (about Example 10)

Below is the graph of the function $P = 80/(t+1)^{0.3}$ in standard coordinates:

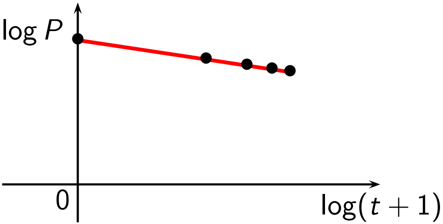
t	$P = 80/(t+1)^{0.3}$
0	80
6	44.62
12	37.06
18	33.072
24	30.458



Comment (cont.d)

Below is the graph of $\log P = \log 80 - 0.3 \log(t+1)$ in a log-log plot:

t	$\log(t+1)$	$\log P = \log 80 - 0.3 \log(t+1)$
0	0	1.903
6	0.845	1.650
12	1.114	1.569
18	1.279	1.519
24	1.398	1.484



Example 11 (Biodiversity):

Some biologists model the number of species S in a fixed area A (such as an island) by the **Species-Area relationship**

$$\log S = \log c + k \log A,$$

where c and k are positive constants that depend on the type of species and habitat.

(a) Solve the equation for S.

(b) Use part **(a)** to show that if $k = 3$ then doubling the area increases the number of species eightfold.