Group: ____________ ____________ ____________ Name: _____________

ma138 Worksheet 13, October 12^{th} 2017

Remember that the determinant of a square matrix A is a number, $\det(A)$, which is non-zero exactly when A has an inverse matrix. One useful property of the determinant is that it is *multiplicative*. What this means is that if A and B are $n \times n$ matrices, then

$$\det(A)\det(B) = \det(AB).$$

This will be useful for problem **1.**

1. Suppose A and B are both $n \times n$ matrices, and A^{-1} and B^{-1} exist. Suppose C is an $n \times n$ matrix which does not have an inverse.

a. Does the matrix AB have an inverse? Why or why not? If it does have an inverse, can you express $(AB)^{-1}$ in terms of A^{-1} and B^{-1}? (**warning:** the answer is not $(AB)^{-1} = A^{-1}B^{-1}$.)

b. Does the matrix BC have an inverse? Why or why not?

c. What is $\det(A^{-1})$? (Use the fact that $AA^{-1} = I_{n\times n}$)

2. Unfortunately, while invertibility and matrix multiplication follow the rules from problem **1.**, the same is not true for matrix addition.

a. Find an example of 2×2 matrices A and B so that A and B are invertible, but $A + B$ is not.

b. Find an example of 2×2 matrices C and D so that neither C nor D are invertible, but $C + D$ is invertible.

c. Find an example of 2×2 matrices E and F so that E is invertible, F is not, and $E + F$ is invertible.

d. Find an example of 2×2 matrices G and H so that G is invertible, H is not, and $G + H$ is not invertible.

3. Let A be the matrix

$$A = \begin{bmatrix} -6 & 5 \\ 7 & -4 \end{bmatrix}$$

Find values x and y that satisfy the matrix equation

$$A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

(It may help if you first re-write the matrix equation so that the left hand side does not depend on x or y.)